Published: Comb.Sci.Tech. 139:1,15-40 1998

EXAMINING THE CASCADE HYPOTHESIS FOR TURBULENT PREMIXED COMBUSTION

A.Y.Klimenko

Department of Mechanical Engineering
The University of Queensland
Brisbane Qld 4072 Australia

e-mail: *klimenko@sun.mech.uq.oz.au*
Facsimile: *(61 - 7) 3365 4799*
Telephone: *(61 - 7) 3365 3670*

Abstract - The cascade hypothesis, which was introduced in application for premixed combustion by Yakhot (1988) and Sivashinsky (1988), is examined against the correlations of the turbulent burning velocity data by Bradley, Lau and Lawes (1992) (BLL data). A new, less restrictive formulation of the cascade hypothesis is suggested. The formulation is shown to be consistent with the BLL data. Various flame characteristics - fractal dimensions, inner cutoffs, main asymptotic regimes - are determined form the BLL data on the basis of the cascade hypothesis.

Key words: premixed combustion, burning velocity, turbulence, cascade hypothesis

1. INTRODUCTION

The cascade hypothesis

Turbulent motions involve random fluctuations of very different characteristic scales. The cascade hypothesis states that the large-scale fluctuations do not interact with the small-scale fluctuations directly. This interaction occurs through a cascade process which involves fluctuations of intermediate scales. The largest eddies break up into smaller eddies whose characteristic scale is comparable with the characteristic scale of the original eddies. The smaller eddies break up into smaller and smaller eddies until the smallest eddies disappear due to the viscous forces. The cascade hypothesis forms the physical basis of the modern theory of turbulence (see Monin and Yaglom, 1973, 1975). The renormalization group methods (see Yakhot, 1988) are closely related to the cascade hypothesis. In these methods, effective averaging over intermediate (and gradually increasing) scales is implemented.

In turbulent premixed combustion, the flame front is affected by turbulent fluctuations of very different scales and it is reasonable to expect that the cascade hypothesis can be used to study the burning velocity. These ideas were originated by Yakhot (1988), who carried out the renormalization group analysis of premixed flames involving so-called ε-expansion of the spectral representation of the transport equations. Sivashinsky (1988) suggested an alternative method of using the cascade hypothesis and modeled the turbulence-flame interactions by the interactions of the flame front with a series of discrete waves. Our goal here is to use the cascade hypothesis as it is, without any further modeling. Of course, this approach will not allow us to obtain any closed equation for the turbulent burning velocity U_T but the cascade hypothesis will be seen to impose certain limitations on U_T. These limitations will be examined in comparison with experimental data.

Turbulent burning velocity

Turbulence enhances the transport processes and increases the burning rate. The turbulent burning velocity U_T depends on the laminar burning velocity U_0, the turbulent integral length scale R_T, the turbulent integral velocity fluctuation scale V_T, the Reynolds number $\mathrm{Re} \equiv V_T R_T / \nu$ and, possibly, some other parameters (such as the Lewis number, Le) which will be discussed later in the paper. The laminar flame thickness δ_0 is not included in the list of determining parameters since δ_0 can be estimated as $\delta_0 = \nu / U_0$ (Bradley, et al., 1992). The viscous (Kolmogorov) velocity and length scales are given by

$$V_K \equiv (\nu\varepsilon_T)^{1/4} = \frac{C_1^{1/4}\ V_T}{\mathrm{Re}^{1/4}} \quad \text{and} \quad R_K \equiv \left(\frac{\nu^3}{\varepsilon_T}\right)^{1/4} = \frac{R_T}{C_1^{1/4}\mathrm{Re}^{3/4}} \tag{1.1}$$

respectively, where $\varepsilon_T = C_1 V_T^3/R_T$ is the average dissipation of energy and C_1 is a constant. One of the possible dimensionless forms of the equation for U_T is given by

$$\frac{U_T}{V_T} = f_{TK}\left(\frac{U_0}{V_K}, \frac{R_T}{R_K}\right) \tag{1.2}$$

where f_{TK} is an unknown function of two variables. The function f_{TK} may also depend on other dimensionless parameters. Specifically, in Section 7, the Lewis number will be included into the list of determining parameters as it is suggested by Bradley et al. (1992). Here, we investigate the dependence of U_T on U_0/V_K and R_T/R_K. The dependence on other dimensionless parameters is implied but not explicitly specified.

Following Yakhot (1988), Sivashinsky (1988), Krestin (1988), Kuznetsov and Sabelnikov (1989), Pocheau (1994) we introduce the effective turbulent burning velocity $U = U(R)$ corresponding to the turbulent fluctuations whose characteristic length scale does not exceed R. The formal definition of U can be given by

$$U(R) = \frac{1}{\pi R^2}\left[\left((c\rho)_+ - (c\rho)_-\right)^{-1} < \int_{|\mathbf{r}-\mathbf{r}_0|\le R} \rho W\, d\mathbf{r}^3 > \right] \tag{1.3}$$

In this equation, we select a sphere so that the flame front passes through its center $\mathbf{r}_0$. The radius of the sphere is R. The chemical source term W of the reactive component c is integrated over the sphere and averaged over realizations. The subscript indices "-" and "+" correspond to the values in the cold and hot regions. The value of $U(R)$ is the averaged burning rate which does not take into account fluctuations whose characteristic scale is greater than R. If $R \to 0$ then the flame front is almost flat inside the sphere and $U \to U_0$ represents the laminar flame speed. If R exceeds the integral length scale of turbulence R_T then $U(R_T) = U_T$ specifies the turbulent burning velocity. The definition of $U(R)$ given by Kuznetsov and Sabelnikov (1989) is most similar to the definition of $U(R)$ used here.

2. MATHEMATICAL FORMULATION OF THE CASCADE HYPOTHESIS

In this section we investigate the parametric representation of the burning velocity inside the inertial interval. The value $U_i = U(R_i)$ is introduced so that R_i belongs to the inertial interval of turbulence $R_K \ll R_i \ll R_T$. The subscript indices 'K' and 'T' indicate that values are related to the Kolmogorov scales and the integral scales respectively. The index 'I' denotes a value which is related to the inertial interval as whole, while the index 'i' emphasizes that the indexed value is selected within the inertial interval range. The value U_i depends on U_0, R_K, V_T, R_T and R_i. According to the Kolmogorov theory, the characteristics of turbulence within the inertial interval are determined by the average energy dissipation ε_T. (This is the basic assumption in Kolmogorov's theory of small-scale turbulence which is also related to the cascade hypothesis - see Monin and Yaglom, 1973, 1975). Thus, R_T and V_T are replaced by ε_T in the list of determining parameters so that U_i depends on U_0, R_K, ε_T and R_i. The dimensionless form of the equation for U_i takes the form

$$\frac{U_i}{V_i} = f_{IK}\left(\frac{U_0}{V_K}, \frac{R_i}{R_K}\right) \tag{2.1}$$

where $V_K = (\varepsilon_T R_K)^{\mu}$ and $V_i = (\varepsilon_T R_i)^{\mu}$ is the characteristic velocity scale which corresponds to the eddies whose characteristic length scale is R_i. Kolmogorov scaling of the inertial interval is given by $\mu = 1/3$.

According to the cascade hypothesis, the interaction between the largest ($\sim R_T$) and the smallest ($\sim R_K$) fluctuations occurs through a series of intermediate scales and R_i is one of these scales. The smallest scales do not affect the largest scales directly. Hence, if all flame and turbulence parameters related to the scale R_i are given then U_T can be determined and we do not need to know the flame characteristics related to the smallest ($\sim R_K$) scales. The turbulent burning velocity U_T is determined by R_T, R_i, U_i and V_i. We expect that the flame fronts with $U(R_i) = U_i$ have similar characteristics of larger ($R > R_i$) scales irrespective of their small-scale structures. The mathematical expression of this understanding of the cascade hypothesis is given by

$$\frac{U_T}{V_T} = f_{TI}\left(\frac{U_i}{V_i}, \frac{R_T}{R_i}\right) \tag{2.2}$$

It is plausible to assume that the dimensionless functions f_{TI} and f_{TK} may have some similarities but these functions are not necessarily the same. The function f_{TK} depends on viscous-scale processes while f_{TI} does not. Note that it would be wrong to deduce from Eq.(2.2) that U_T does not depend on U_0. Indeed, U_T depends on U_i while U_i is expected to be a strong function of U_0.

Combining the discussions led to equation (2.1) and (2.2) we obtain

$$\frac{U_2}{V_2} = f_{II}\left(\frac{U_1}{V_1}, \frac{R_2}{R_1}\right) \tag{2.3}$$

where $U_1 = U(R_1)$, $U_2 = U(R_2)$; R_1 and R_2 are arbitrary scales within the inertial interval: $R_K \ll R_1 \le R_2 \ll R_T$. The function f_{II}, which is unknown function of two variables, determines the flame front behavior within the inertial interval. Equation (2.3) represents the mathematical interpretation of the cascade hypothesis and will be used in the next section. This equation implicitly imposes certain restrictions on Eq.(1.2).

Pocheau (1994) considered the representation of the burning velocity which, if we use the notations adopted here, may be written in the form $U_2/U_1 = f_P\left(V_2/U_1, R_2, \Delta R\right)$, where $\Delta R = R_2 - R_1$. Although this equation has some similarities with Eq.(2.3), these equations are not functionally identical. In his analysis, Pocheau (1994) used the "covariance by dilatation" assumption which requires certain invariance of the function f_P with respect to changes of R_2 and ΔR. This assumption is very restrictive and it is not used in the present work.

3. ASSOCIATIVE CONSISTENCY

In this section we consider certain constraints which must be satisfied by the function f_{II}. These constraints are determined by consistency requirements and do not constitute any additional assumption. Equation (2.3) can be rewritten in the form

$$a_2 = \varphi\left(a_1, b_2 - b_1\right) \tag{3.1}$$

where

$$a \equiv \ln\left(\frac{U}{V}\right), \quad b \equiv \ln(R), \quad \varphi(a,b) \equiv \ln\left(f_{II}(e^{a}, e^{b})\right) \tag{3.2}$$

and the indices of a and b correspond to the indices of U, V and R. It is quite obvious that the function φ must satisfy

$$a_1 = \varphi\left(a_1, 0\right) \tag{3.3}$$

Another restriction is given by the associative constraint. Let us consider three different scales R_1, R_2 and R_3 within the inertial interval $R_K \ll R_1 \le R_2 \le R_3 \ll R_T$. The equation

$$a_3 = \varphi\left(a_1, \Delta b_1 + \Delta b_2\right) = \varphi\left(\varphi(a_1, \Delta b_1), \Delta b_2\right) \tag{3.4}$$

where $\Delta b_1 \equiv b_2 - b_1$ and $\Delta b_2 \equiv b_3 - b_2$, can be derived by substituting $\varphi(a_1, \Delta b_1)$ for a_2 in $a_3 = \varphi(a_2, \Delta b_2)$. The structure of Eq.(3.4) is quite similar to the structure of the identity $b_3 = b_1 + (\Delta b_1 + \Delta b_2) = (b_1 + \Delta b_1) + \Delta b_2$: these equations represent the associative consistency. The value of a_3, predicted using the one-step $(\Delta b_1 + \Delta b_2)$ representation, and the value of a_3, predicted using two steps Δb_1 and Δb_2, must be the same.

Equation (3.4) is differentiated with respect to Δb_1

$$\frac{da_3}{d(\Delta b_1)} = \varphi'_b\left(a_1, \Delta b_1 + \Delta b_2\right) = \varphi'_a\left(\varphi(a_1, \Delta b_1), \Delta b_2\right) \cdot \varphi'_b(a_1, \Delta b_1) \qquad (3.5)$$

where

$$\varphi'_b \equiv \frac{\partial \varphi(a,b)}{\partial b} \qquad \varphi'_a \equiv \frac{\partial \varphi(a,b)}{\partial a}$$

Substituting $\Delta b_1 = 0$, $\Delta b_2 = b$ and $a_1 = a$ into Eq.(3.5) and taking into account Eq.(3.3), we obtain

$$\varphi'_b(a,b) = \varphi'_a(a,b) \cdot \psi(a) \qquad (3.6)$$

where

$$\psi(a) \equiv \varphi'_b(a,0)$$

Within the inertial interval, Eq.(3.6) is valid for any b and a. Equation (3.6) can be integrated to yield $\varphi(a,b) = \phi\left(F_I(a) + b\right)$, where $dF_I/da = 1/\psi(a)$ and ϕ is an arbitrary function. Since $\varphi(a,0) = a$ for any a, the final form of the function is given by $\varphi(a,b) = F_I^{-1}\left(F_I(a) + b\right)$ where F_I^{-1} is the inverse function of F_I: $F_I^{-1}\left(F_I(a)\right) \equiv a$. Equation (3.1) takes the form

$$a_2 = F_I^{-1}\left(F_I(a_1) + (b_2 - b_1)\right) \qquad (3.7)$$

Equation (3.7) can be also written in the form $F_I(a_1) - b_1 = F_I(a_2) - b_2$. Let us introduce $B \equiv F_I(a_i) - b_i$. We stress that the value B is constant and it does not depend on selection of b_i when b_i belongs to the inertial interval. The equation for a_i can be written as

$$a_i = F_I^{-1}\left(B + b_i\right); \qquad \frac{da_i}{db_i} = \psi(a_i) = \frac{1}{F'_I(a_i)} \qquad (3.8)$$

where $F'_I \equiv dF_I/da$. Note that the constant B depends on the units used to measure R. Hence, the constant B is not universal and its functional representation needs to be determined.

If the intensity of turbulence is very small $U/V \to \infty$, it is reasonable to expect that turbulence does not affect the flame front and $U_2 \to U_1$. This limit corresponds to $a_2 \to a_1 - \ln\left(V_2/V_1\right) = a_1 - \mu(b_2 - b_1)$ as $a_1, a_2 \to \infty$ and it is consistent with Eq.(3.7). The limiting form of the function F_I is given by $F_I(a) \to -a/\mu$ + const.

4. MATCHING WITH THE SMALLEST AND LARGEST SCALES

Equation (3.8), which is valid only within the inertial interval, should be matched with the functional representations of the burning velocities outside the inertial interval. This relates U_T to U_0 and determines the constant B.

The smallest scales are considered first. Equation (2.1) can be written in the form $a_i = \varphi_K(a_K, b_i - b_K)$ where $a_K \equiv \ln(U_0/V_K)$, $b_K \equiv \ln(R_K)$ and φ_K is related to f_{IK} by Eq.(3.2). The asymptote $\overset{+}{\varphi}_K$ of the function φ_K as $R_i/R_K \to \infty$ must be consistent with Eq.(3.8) that is $\overset{+}{\varphi}_K(a_K, b_i - b_K) = F_I^{-1}(B + b_i)$. This equation can be written in the form

$$F_I\left(\overset{+}{\varphi}_K(a_K, b_i - b_K)\right) = B + b_i = B + b_K + (b_i - b_K) \tag{4.1}$$

Since the constant B does not depend on b_i, the sum $B + b_K$ is also independent of b_i. This gives the equation for the constant B

$$B = F_K(a_K) - b_K \tag{4.2}$$

where $F_K(a_K) \equiv F_I\left(\overset{+}{\varphi}_K(a_K, b)\right) - b$. Note that F_K does not depend on b. The function $F_K(a)$ does not necessarily coincide with $F_I(a)$.

Matching with the largest scales is quite similar to matching with the smallest scales. We rewrite Eq.(2.2) in the form $a_T = \varphi_T(a_i, b_T - b_i)$ and consider the asymptote $\varphi_T \to \overset{-}{\varphi}_T$ as $R_i/R_T \to 0$. Matching of $\overset{-}{\varphi}_T$ and Eq.(3.8) yields $\overset{-}{\varphi}_T(a_T, b_T - b_i) = F_I^{-1}(B + b_i)$. Finally, after repeating all steps in the derivation of Eq.(4.2), we obtain the equation $B = F_T(a_T) - b_T$ where $F_T(a_T) \equiv F_I\left(\overset{-}{\varphi}_T(a_T, b)\right) - b$ does not depend on b. Combining of the result with Eqs.(3.8) and (4.2) yields

$$B = F_K(a_K) - b_K = F_I(a_i) - b_i = F_T(a_T) - b_T \tag{4.3}$$

The functions F_T, F_K and F_I determine scaling of the turbulent burning velocity $U(R)$. These functions may have some common features but they do not necessarily coincide with each other. The function F_I which characterizes the inertial interval is expected to be universal. The function F_K is determined by viscous-scale processes and F_K may depend on dimensionless parameters characterizing the smallest scales. An important example of such parameter is given by the Lewis number (Le). The function F_K describes the interaction of a laminar flame front with turbulence of the smallest scales while the function F_I describes the interaction of the turbulent fluctuations with the flame front which has been already wrinkled by the fluctuations of smaller scales. This difference between F_I and F_K will be shown to be quite significant. The function F_T depends on the dimensionless parameters characterizing the largest scales. The largest scales in turbulence are not universal and, in a rigorous approach, the parameters characterizing the geometry of the flow should be treated as determining parameters for F_T while the function F_I does not depend on these parameters. On other hand, both functions F_T and F_I describe the interaction of the wrinkled flame with turbulence and must have more common features than F_K and F_I. It is reasonable to assume that $F_T(a) = F_I(a)$. Neglecting the non-universality of the large-scale fluctuations is a common assumption in turbulent combustion theories and it will be used here for some estimations. The instantaneous position of the flame front in a non-homogeneous flow is constantly varied by large-scale fluctuations. This may require averaging of the characteristics of turbulence over the front positions. Averaged characteristics can be expected to have higher degree of universality than the local characteristics of turbulence (see Klimenko et al., 1995). Note that both functions F_T, F_K and F_I may indicate dependence on one additional parameter - the density ratio $\theta \equiv \rho_+/\rho_-$.

5. THE EQUATION FOR TURBULENT BURNING VELOCITY

As it follows from Eq.(4.3), the value of a_T is given by the equation

$$a_T = F_T^{-1}\left(F_K(a_K) + b_T - b_K\right) \tag{5.1}$$

where F_T^{-1} is the inverse function of F_T. Using the definitions introduced in Eq.(3.2), we can rewrite Eq.(5.1) in the form

$$\frac{\Phi_T\left(\frac{U_T}{V_T}\right)}{R_T} = \frac{\Phi_K\left(\frac{U_0}{V_K}\right)}{R_K} \quad \text{or} \quad \frac{U_T}{V_T} = \Phi_T^{-1}\left(\frac{R_T}{R_K}\,\Phi_K\left(\frac{U_0}{V_K}\right)\right) \tag{5.2}$$

where

$$\Phi(\alpha) \equiv \exp\left(F\left(\ln(\alpha)\right)\right) \tag{5.3}$$

and the subscript indices of Φ correspond to the indices of F. Equation (5.2) specifies the structure of the function f_{TK} in Eq.(1.2) which corresponds to the cascade hypothesis. The cascade hypothesis effectively reduces some degrees of freedom in choosing f_{TK}. The function f_{TK} is a function of two variables while Eq.(5.2) is determined by two one-variable functions Φ_T and Φ_K. (In general, a two-variable function is determined by an infinite number of one-variable functions.)

In analysis of the the burning velocity, the largest $\sim R_T$ and the smallest $\sim R_K$ scales are often formally treated as part of the inertial interval. This corresponds to the assumption $\Phi_T(\alpha) = \mathrm{const}\cdot\Phi_K(\alpha)$ for any α The constant allows for introduction of the Kolmogorov scale and macroscale which differ from the conventional definitions by constant factors. This assumption makes Eq.(5.2) too restrictive and it will be shown that in this case Eq.(5.2) can not match the experimental data. It is easy to show that the traditional representation of turbulent burning velocity U_T/U_0 = $f_B\left(V_T/U_0\right)$ (see Bradley, 1992) is consistent with Eq.(5.2) and corresponds to $f_B(\alpha) = \alpha\,\Phi_T^{-1}(C_1\alpha^3)$, $\Phi_K(\alpha) = \alpha^{-3}$. However, this representation does not comply with the assumption $\Phi_T(\alpha) = \mathrm{const}\cdot\Phi_K(\alpha)$.

Sivashinsky's (1988) equation for the burning velocity can be written in the form

$$\ln\left(\frac{U_T}{U_0}\right) = \lambda\int_0^{b_T} \ln\left[S\left(\frac{V}{U}\right)\right] db \tag{5.4}$$

where $\lambda = 2/3$; $-b$ is substituted for the logarithm of the wave number and the function S has been determined as the solution of a modeling problem. The turbulent burning velocity U is the result of interaction of a passively convected laminar flame front, whose laminar propagation velocity is U_0, with one-scale homogeneous isotropic field, whose intensity is V. Sivashinsky (1988) introduced the function S by the equation

$$\frac{U}{U_0} = S\left(\frac{V}{U_0}\right) \tag{5.5}$$

and evaluated this function for two-dimensional periodic flow field as

$$S\left(\frac{V}{U}\right) \approx \min\left\{1 + \beta^{-1},\quad 1 + \frac{(V/U)^2\left((V/U)^2\beta^2+1\right)}{4\left((V/U)^2\beta^2-1\right)^2}\right\} \tag{5.6}$$

This approximation depends on the inertial interval parameter β introduced by Zeldovich (1982) as $\beta \equiv \omega k^{-1} V^{-1}$ where ω is the frequency and k is the wave number. After differentiation, Eq.(5.4) takes the form

$$\frac{d\ \ln(U)}{db} = \lambda \ln\left(S\left(\frac{V}{U}\right)\right) \tag{5.7}$$

Let us compare this equation with scaling within the inertial interval given by Eq.(3.8)

$$\frac{d\ \ln(U)}{db} = \frac{d(a+\ln(V))}{db} = \psi(a) + \mu = \frac{1}{F'_{\mathrm{I}}(a)} + \mu \tag{5.8}$$

where $\mu \equiv d\ln(V)/db = 1/3$. It is easy to see that both equations are, as it is expected, consistent with each other and

$$S_{\mathrm{I}}\left(e^{-a}\right) = \exp\left(\frac{1}{\lambda}\left(\frac{1}{F'_{\mathrm{I}}(a)} + \mu\right)\right) \tag{5.9}$$

There are, however, some differences. First, the function F_{I} is not bound by any modeling assumptions and will be found by analyzing the experimental data. Second, Eq.(5.2), which determined by two functions (Φ_K and Φ_T), has more degrees of freedom than Eq.(5.7) determined only by one function S. The index 'I' is used to distinguish the function S specified by Eq.(5.6) and the function S_{I} specified by Eq.(5.9).

6. THE CASCADE HYPOTHESIS AND FRACTAL DIMENSION

In this section, we follow Peters (1986), Gouldin (1987) and Bradley (1992) and consider the fractal structure of premixed flames assuming that the flame front is a wrinkled surface with the laminar burning velocity U_0. The fractal dimension will be related to the scaling functions introduced in previous sections. We consider the section of the flame front inside the sphere of radius R which is chosen so that the sphere is chosen so the flame front passes through its center. Its fractal dimension is introduced as

$$D = \frac{\ln\left(N(R,R_0)\right)}{\ln(R/R_0)} \equiv \frac{\ln\left(N(R,R_0)\right)}{b - b_0} \tag{6.1}$$

where $b \equiv \ln(R)$, $b_0 \equiv \ln(R_0)$ and $N(R_2,R_1)$ is the minimal number of spheres of radius R_1 (or other volumes whose size does not exceed R_1) required to cover the surface inside the sphere of radius R_2. The mathematical definition of the fractal dimension (Kolmogorov fractal capacity) involves the limit $R_0 \to 0$ in Eq.(6.1). Unlike mathematical fractals, the flame fronts are wrinkled only at scales exceeding certain scale R_0 which is called the inner cutoff. The flame front is flat for a scale smaller than R_0. Thus, the inner cutoff scale R_0 is fixed in the definition of the fractal dimension in Eq.(6.1). Note that the fractal dimension D, as it is introduced here, can be different for different R and $D = D(R)$ in Eq.(6.1).

The average area of the flame surface inside the sphere of radius R is denoted as A_R. Estimating the surface inside each sphere R_0 as πR_0^2 and multiplying by the number of spheres required, it is easy to conclude that the area of a surface whose fractal dimension is D and inner cutoff is R_0 is given by

$$A_R \sim \pi R_0^2 \left(\frac{R}{R_0}\right)^D \qquad (6.2)$$

The flame front area A_R inside the sphere of radius R is proportional to the flame speed $U(R)$

$$U(R) = U_0 \frac{A_R}{\pi R^2} \sim U_0 \left(\frac{R}{R_0}\right)^{D-2} \qquad (6.3)$$

Equation (6.3) can be derived from Eq.(1.3) if we note that the term in the square brackets is $U_0 A_R$.

It is convenient for our purposes to introduce the new value

$$D^* \equiv \frac{d(b-b_0)D}{db} \quad ; \qquad D(b) = \frac{1}{b-b_0} \int_{b_0}^{b} D^*(b^\circ)\, db^\circ \qquad (6.4)$$

which is directly related to the fractal dimension D. If D^* is constant then $D = D^*$. Taking the logarithm of both sides of Eq.(6.3) we obtain $\ln(U/U_0) = (D-2)(b-b_0) + C_A$ where C_A is the logarithm of the constant factor in Eq.(6.3). This equation is differentiated with respect to b and the derivative $d\ln(U)/db$ is determined from Eq.(3.8). The resulting equation

$$D^* = \frac{1}{F'_I(a)} + 2 + \mu = \psi(a) + 2 + \mu \qquad (6.5)$$

$(\mu = 1/3)$ links D^* and the scaling functions. This equation specifies the fractal dimension of the flame front which is scaled according to the cascade hypothesis. The fractal dimension D given by Eqs.(6.4) and (6.5) is not necessarily a constant (although, constant D^* and D are also in agreement with the cascade hypothesis).

It is important to emphasize that we do not assume here that the fractal dimension of the flame front is constant. The values of D and D^* may depend on R and be different for different scales. The value $D(R)$ characterizes the average fractal dimension in the range of scales $[R_0,R]$ while $D^*(R)$ is a characteristic of scale R. If D and D^* are not constant and the flame front is not a fractal in strict mathematical sense of this term, the concept of fractal dimension can still be useful. Equations (6.1)-(6.5) represent definitions of the fractal dimension for the flame fronts which may not be pure mathematical fractals. However, it is reasonable to expect that the flame fronts exhibit some properties of the fractal surfaces and the value D, measured with proper averaging, is a smooth function of R. The fractal dimension is related to other scaling functions used here by Eq.(6.5) and will be determined from experimental data for several asymptotic regimes of flame propagation.

7. DETERMINING THE SCALING FUNCTIONS

Equation (5.2) is the mathematical expression of the cascade hypothesis applied to premixed combustion. This equation is not a closed theoretical formula for burning velocity but it effectively reduces the degree of freedom in choosing f_{TK} in Eq.(1.2) (i.e. not every function f_{TK} can be represented by Eq.(5.2)). The burning velocity in Eq.(5.2) is specified by the two scaling functions $\Phi_K(a)$ and $\Phi_T(a)$ (or by $F_K(a)$ and $F_T(a)$) where $\Phi_K(a)$ is determined by the viscous-scale processes and $\Phi_T(a)$ is determined by the integral-scale processes. Our goal in this section is to find out if it is possible to represent the known experimental data in the form of Eq.(5.2). A positive answer would be a confirmation of the cascade hypothesis.

The correlations of the experimental data by Bradley et al. (1992) provide a wide range for the parameters U_0/V_K and R_T/R_K. These data, which are referred to here as the BLL (Bradley, Lau and Lawes) data, represent the burning velocity measurements using the two dimensionless parameters $K\text{Le}$ and Re/Le^2 where K is the Karlovits stretch factor and Le is the Lewis number. The BLL data comprise more than 1500 experimental points (basically all known measurements) and it is the most comprehensive systematization of the burning velocity data available. The original experimental points have certain scattering over the parametric curves. This scattering is induced by measurement errors, by uncertainties in estimations and, may be, by parameters which affect U_T but are not listed as determining parameters. The BLL data, involving a large number of experimental points, are effective averages which represent a realistic dependence of the burning velocity U_T on $K\text{Le}$ and Re/Le^2.

Dependence on the Lewis number

It was shown by Bradley et al.(1992) that the dimensionless parameters $K\,\mathrm{Le}$ and $\mathrm{Re}/\mathrm{Le}^2$ represent the best choice for parametric representation of the available experimental data. Under this assumption, Eq.(1.2) takes the form

$$\frac{U_T}{V_T} = f_{TK}\left(\frac{U_0}{V_K}\,\mathrm{Le}^{-1/2},\ \ \frac{R_T}{R_K}\,\mathrm{Le}^{-3/2} \right) \qquad (7.1)$$

The Karlovits stretch factor K can be found from the isotropic turbulence equation $K = C_2\,(V_T/U_0)^2\,\mathrm{Re}^{-1/2}$ where $C_2 \approx 0.157$. The dimensionless parameters used here can be determined from the equations

$$\frac{R_T}{R_K}\mathrm{Le}^{-3/2} = C_1^{1/4}\left(\frac{\mathrm{Re}}{\mathrm{Le}^2}\right)^{3/4}$$
$$\frac{U_0}{V_K}\mathrm{Le}^{-1/2} = \frac{U_0}{V_T}\left(\frac{\mathrm{Re}}{C_1\mathrm{Le}^2}\right)^{1/4} = C_3\,(K\,\mathrm{Le})^{-1/2} \qquad (7.2)$$

where $C_1 \approx 0.37$ and $C_3 \equiv C_2^{1/2} C_1^{-1/4} \approx 15^{-1/4} \approx 0.508$. These approximations are chosen according to Bradley et al.(1992).

It was noted above that the functions Φ_K and F_K may depend on other dimensionless parameters characterizing the smallest scales. Such a dependence was implied but was not considered in Eqs.(4.3) and (5.2). We assume here that the dependence of Φ_K and F_K on Le is given by

$$F_K(a,\mathrm{Le}) = \overset{\circ}{F}_K\Big(a - \ln(\mathrm{Le})/2\Big) - \frac{3}{2}\ln(\mathrm{Le}) \qquad (7.3)$$

$$\Phi_K(\alpha,\,\mathrm{Le}) = \mathrm{Le}^{-3/2}\,\overset{\circ}{\Phi}_K(\alpha\,\mathrm{Le}^{-1/2}) \qquad (7.4)$$

where the functions $\overset{\circ}{F}_K$ and $\overset{\circ}{\Phi}_K$ are universal and related to each other by Eq.(5.3). This representation is consistent with Eq.(7.1). Equations (4.3) and (5.2) take the forms

$$a_T = F_T^{-1}\Big(F_K(\overset{\circ}{a}_K) + b_T - \overset{\circ}{b}_K\Big) \qquad (7.5)$$

$$\frac{U_T}{V_T} = \Phi_T^{-1}\left(\frac{R_T}{R_K}\,\mathrm{Le}^{-3/2}\,\overset{\circ}{\Phi}_K\left(\frac{U_0}{V_K}\,\mathrm{Le}^{-1/2}\right)\right) \qquad (7.6)$$

where $\overset{\circ}{a}_K \equiv \ln\left(\frac{U_0}{V_K}\right) - \ln(\mathrm{Le})/2$; $\overset{\circ}{b}_K \equiv \ln(R_K) + \frac{3}{2}\ln(\mathrm{Le})$. These functions formulate the cascade hypothesis for the BLL data. Equation (7.6) is consistent with both the cascade hypothesis and the representation of the BLL data. It should be noted that Eq.(7.6) is more restrictive than the original representation of the BLL data by Eq.(7.1) and not every function f_{TK} can be represented in the form of Eq.(7.6).

Scaling functions and main asymptotes

The approximations for the functions $\overset{\circ}{F}_K(a)$ and $F_T(a)$ are given in Table I. These approximations were chosen so that Eq.(7.6) has the same asymptotic structure as the BLL data. It is convenient to use different representations in the regions $\overset{\circ}{a}_K \le h_a$ and $\overset{\circ}{a}_K \ge h_a$. The functions are determined by 17 constants. Four of these constants are bounded by the condition of matching the functions at $\overset{\circ}{a}_K = h_a$. One of the constants does not affect the data and we can put $\bar{h}_{K1} = 0$ without loss of generality.

The constants have been determined numerically so that Eq.(7.6) gives the best fit to the BLL data. The values of the constants are given in Table II. The relative approximation error does not exceed 10% and the average value of the relative error is less than 4%. The relative error was determined as $\left|U_T - U_T^*\right|/U_T$ where U_T represents the data and U_T^* is the approximation. Equation (7.6) provides reasonably good approximation of the BLL data.

The functions $1/F'_K$ and $1/F'_T$, where $F'_K \equiv d\overset{\circ}{F}_K(a)/da$ and $F'_T \equiv dF_T(a)/da$, are shown on Figure 1. It is easy to see that $1/F'_K$ and $1/F'_T$ are almost zero in the interval $0 < a < 1.5$. This singular behavior will be discussed in the next section. Another feature of the functions $F_K(a)$ and $F_T(a)$ is that their derivatives of are always non-positive. As it was discussed in Section 4 we can neglect the difference between scaling of the inertial interval and scaling of the largest scales and assume that $F_I(a) = F_T(a)$. The value D^* is determined by Eq.(3.8): $D^* = 7/3 + 1/F'_T$. According to Figure I, D^* varies within the limits $2 < D^* \le 7/3$. The derivative $1/F'_K$ does not have a simple physical interpretation and is given for comparison.

The function $S_I\left(V/U\right)$ determined from Eq.(5.9) is compared on Figure 2 with the theoretical prediction of S in Eq.(5.6). The value $\beta = 1.4$ is used in Eq.(5.6). Agreement with the function S determined from the BLL data is remarkable for $V/U > 0.5$ but the value of S is evidently underestimated for small V/U. The function S_I slightly decreases as for small V/U but $S_I \sim 1.5$ as $V/U \to 0$. The asymptote $S \sim 1 + C_S (V/U)^2 \to 1$ as $V/U \to 0$ $(C_S = \text{const})$ was obtained by Sivashinsky (1988) as the result of relatively strict asymptotic analysis. In his asymptotic analysis, the density variations were neglected and this points to the area where the explanation of the differences between S and S_I should be sought. If $U \gg V$, then $S \approx 1$ and the passively convected interface is not strongly affected by turbulence.

The burning velocity results are presented on Figures 3 and 4 which show different asymptotic regimes by selecting different coordinates X. The approximation is shifted in Y-direction. The turbulent burning velocity dependence on U_0/V_K and Re (we assume $Le = 1$) and clearly indicate two asymptotic regions: fast flames and slow flames. The existence of the third asymptote (very fast flames) for $U_0/V_K>8$ is not so clear. The asymptotes determined from the formulae in Tables I and II are listed in Table III where $\overset{\circ}{b} \equiv b_T - \overset{\circ}{b}_K$. If the Re number is moderate, the inertial interval does not exist and the largest scales should be matched with the smallest scales directly. Equation (7.6) which is based on the cascade hypothesis is, however, an acceptable approximation for the whole range of Reynolds numbers in the BLL data. The BLL data do not clearly indicate any special region or behavior which is related to moderate Reynolds numbers.

8. FAST FLAMES

Fast flames are characterized by $K < 1$ and relatively large values of U/V. There is no question that these flames correspond to the flamesheet (or flamelet) regime (see Williams, 1985; Pope, 1987). The fast flames can be treated as laminar flames wrinkled by turbulence. As it follows from Table III, the function

$$\frac{U_T}{V_T} = \gamma_1 \left(\frac{U_0}{V_K} \right)^{\gamma_2} \mathrm{Le}^{-\gamma_2/2}; \qquad \gamma_1 \approx 1.3,\ \gamma_2 \approx 0.6 \tag{8.1}$$

is a good approximation for the fast flames. The existence of this asymptote was pointed out by Bradley et al. (1992). The ratio U_T/V_T does not indicate any significant dependence on R_T/R_K and Re. The convergence of the curves corresponding to different $\mathrm{Re}/\mathrm{Le}^2$ is very clear on Figure 3. The features of the fast flames are analyzed below assuming, for simplicity, that $Le = 1$.

In the fast flames region, a_T does not depend on $b_T - b_K$ since $|F'_T| \to \infty$ and $|F'_K| \to \infty$ and Eqs.(4.3) and (7.3) become degenerate

$$a_T = F_T^{-1}\left(\overset{\circ}{F}_K (a_K) \right) = F_* (a_K) \qquad \text{and} \qquad \frac{U_T}{V_T} = \Phi_* \left(\frac{U_0}{V_K} \right) \tag{8.2}$$

where Φ_* is related to F_* by Eq.(5.3). It is obvious that the functions $F_T(a)$ and $\overset{\circ}{F}_K(a)$ are not the same. Indeed, if $F_T(a) = \overset{\circ}{F}_K(a)+\mathrm{const}$ then $U_T/V_T \sim U_0/V_K$ and this is not in agreement with Eq.(8.1). In the region where $|F'_T| \to \infty$ and $|F'_K| \to \infty$ the functions $F_T(a)$ and $\overset{\circ}{F}_K(a)$ are not unique: only the function $F_*(a)$ is uniquely determined by the BLL data. This non-uniqueness in the choice of the functions $F_T(a)$ and $\overset{\circ}{F}_K(a)$ does not affect the properties of the fast flames discussed here.

The fractal dimension

If $|F'_T| \to \infty$ and $|F'_K| \to \infty$ then, as it follows from Eq.(4.3), $|F'_I| \to \infty$. The value D^* determined by Eq.(6.5) tends to the constant value, 7/3, as $1/F'_I \to 0$. Hence, the fractal dimension of the fast flames is constant

$$D = 2\frac{1}{3} \tag{8.3}$$

Equation (8.3) means that, within the inertial interval, the effective burning velocity U has the same scaling with the fluctuation velocity V and the ratio U/V is constant. This hypothesis was clearly formulated by Krestin (1988) who pointed out that, because of the density difference $\theta \neq 1$, the flame front is not a passively convected interface and, thus, it should be dynamically coupled with turbulence. The geometrical properties of the fast flames (determined by the fractal dimension) do not depend on the ratio U/V. Turbulence is efficient in increasing the propagation speed (larger values U correspond to larger R) of the fast flames even if $U \gg V$ and this efficiency is difficult to explain if the flame front is considered as a passively convected interface. The combination of the density difference at the flame front and the random change of the flame orientation generates hydrodynamic fluctuations. These fluctuations can be responsible for the rapid increase of the flame speed. Their intensity V^* should be estimated as $V^* \sim U$. Otherwise, weak ($V^* \ll U$) fluctuations can not significantly affect the flame front. The process of the rapid increase of the propagation speed is controlled by the vorticity of the intrinsic turbulence and U has the same scaling as V within the inertial interval.

The inner cutoff and the transitional region

The flame front is almost flat for the scales smaller than the inner cutoff R_0 and $U(R) \approx U_0$ for $R \leq R_0$. If the fractal dimension of the flame front is $D = 7/3$ then, according to Eq.(6.3), $U_T/V_T \sim U_0/V_0$ where $V_0 \equiv V(R_0)$. The assumption that the inner cutoff coincides with the Kolmogorov scale $R_0 \sim R_K$ is quite plausible but it yields the estimation

$$U_T/V_T \sim U_0/V_K = R_K/\delta_0 \tag{8.4}$$

which is not in agreement with Eq.(8.1). Let us determine the value of the effective inner cutoff which is consistent with the BLL data and Eq.(8.1). As it follows from the substitution of $R = R_T$ and $U = U_T$ into Eq.(6.3), the effective inner cutoff $R_0 \sim R_T(U_0/U_T)^3$ which corresponds to Eq.(8.1) is given by

$$R_0 \sim R_K \left(\frac{U_0}{V_K} \right)^{\gamma_0}; \qquad \gamma_0 = 3(1-\gamma_2) \approx 1.2 \tag{8.5}$$

The effective inner cutoff is larger than the Kolmogorov scale for the fast flames with $U_0 \gg V_K$. The flame is intensively wrinkled in much of the inertial interval. The density difference across the flame front, combined with its random wrinkling, generates additional hydrodynamic fluctuations. These fluctuations provide rapid increase of the flame speed so that the ratio U/V and the fractal dimension D (=7/3) remain constant for gradually increasing R. Flames of this type can be called as fully-developed turbulent flames. This region does not, however, reach the viscous scales. At the scale R_0 which (presumably) can be estimated by Eq.(8.5) the fame front becomes flatter. It generates smaller fluctuations and its interactions with turbulence are not as efficient as at larger scales. In this region, which involves the smallest scales in turbulence, the flame front experiences a transition from the undisturbed flames to the fully-developed flames. During this process, the ratio U/V deviates from U_0/V_K to $\sim (U_0/V_K)^{\gamma_2}$, $\gamma_2 \approx 0.6$. The range of R which corresponds to this process can be called as the transitional region. The BLL data clearly indicate that the turbulence interactions with the flames which have been wrinkled by smaller fluctuations (i.e. fully-developed flames) are different from the turbulence interactions with the undisturbed flames. For this reason, we can not assume that $F_T(a) = F_K(a)$ + const.

Let us consider the behavior of the flame fronts in the transitional region for small V/U as it is predicted by the function $S(V/U)$. According to Sivashinsky's (1988) asymptotic analysis, $(S-1) \sim (V/U)^2 \to 0$ as $V/U \to 0$. That is, if $V_K/U_0 \ll 1$, the turbulent velocity specified by Eq.(5.5) remains almost unchanged: $U/U_0 = 1$. Significant increase of the flame speed is possible only at scale R where V is sufficiently large so that $V/U_0 \sim 1$ and S is noticeably greater than 1. This is similar to Peters's (1986) concept of the Gibson scale R_G. The Gibson scale is determined by the condition $V(R_G) \sim U_0$. Hence, $R_G \sim R_K(U_0/V_K)^3$. The assumption $R_0 \sim R_G$ combined with $D = 7/3$ yields $U_T/V_T \sim U_0/V_G \to$ const as $V_K/U_0 \to 0$ (where $V_G \equiv V(R_G) \sim U_0$) as opposed to Eq.(8.1). Sivashinsky's function $S(V/U)$ and Peters's concept of the Gibson scale correctly specifies the tendency of the the ratio R_0/R_K to rise as U_0/V_K increases. The comparison with the BLL data indicates that the tendency is overpredicted by these theories.

The flame instability.

Landau (1944) pointed out that the laminar flame fronts acting as the surfaces separating mixtures with different densities are unstable. The comprehensive analysis of the stability of premixed flames which takes into account many other factors can be found in Clavin (1975). The flame instability can potentially wrinkle the flame front at the scales smaller than R_K and further increase the turbulent propagation velocity. In this case, it is plausible to assume that the inner cutoff scale is determined by the flame thickness $R_0 \sim \delta_0$. If $D = 7/3$ everywhere within the range $\delta_0 < R < R_T$ then, according to Eqs.(6.3) and (8.5), $U_T/V_T \sim (U_0/V_K)(R_K/\delta_0)^{1/3} = (U_0/V_K)^{4/3}$. Certainly, the exponent 4/3 is not consistent with the experimental data.

Alternatively, we can assume that the flame instability fully controls the flame front in the region $R<R_G$ where the laminar burning velocity is greater than turbulent fluctuations $U_0>V(R)$. Let us follow Kuznetsov and Sabelnikov (1989) and assess the essentially non-linear and stochastic stage of evolution of the flame instabilities. Initially, the flow is laminar so this problem has only two main determining parameters: δ_0 and U_0. The turbulent propagation velocity is given by the equation $U = U_0 f_0(R/\delta_0)$. The function f_0 may depend on other parameters which characterize the flame instability. Kuznetsov and Sabelnikov (1989) advanced the argument (which has some similarities with the theory of logarithmic boundary layers) that the function f_0 is logarithmic for $R/\delta_0 \gg 1$, that is $U/U_0 \sim \ln(R/\delta_0) + \kappa$ where κ is a constant. The fractal dimension of this flame, given by Eq.(6.3), is $D = 2+\ln(b_0)/b_0 \approx 2$ where $b_0 \equiv \ln(R/\delta_0)$. The flame speed increase due to the flame instability is so slow that the fractal dimension of the flame is almost 2. More efficient wrinkling needs the vorticity of intrinsic turbulence. Thus, the length scale of the smallest wrinkles is of the order of δ_0 but $D \approx 2$ in the range $\delta_0 < R < R_G$. The effective value of the inner cutoff remains $R_0 \approx R_G$ and this is not supported by the BLL data. The flame instability can, however, increase the effective laminar burning velocity: $U_0^* \approx \kappa U_0$.

Flame generated turbulence

Let us consider the fast flame interactions with turbulence in the transitional region. It is likely that the fast flames are affected by the smallest turbulence fluctuations stronger than passively convected interfaces because of the fluctuations generated by the flame. In the transitional region the generation of fluctuations is not very strong because the flame front is not sufficiently wrinkled. The fluctuations $V^* \sim U$ are generated at larger scales where the flame front is fully-developed and are then transferred to the dissipation scales where the flame front is almost flat. These fluctuations increase the level of turbulence at the dissipation scales and accelerate the flame front. The Kolmogorov scale can be estimated by the equation $R_K \sim R_T^{1/4}(\nu/V_T)^{3/4}$. (We use Eq.(1.1) and estimate the dissipation of energy as $\varepsilon_T \sim V_T^3/R_T$.) The flame generated fluctuations reduce the effective value of the Kolmogorov scale $R_K^* \sim R_T^{1/4}(\nu/V_T^*)^{3/4} \sim R_K(V_T/V_T^*)^{3/4}$. The value of the Kolmogorov velocity is also affected by the generated fluctuations $V_K^* = \nu/R_K^* \sim (V_T^*/V_T)^{3/4}\nu/R_K = V_K(V_T^*/V_T)^{3/4}$. We assume that the equation (8.4) is valid but we substitute the effective value V_K^* for V_K (the substitution of R_K^* for R_K yields the same result). Equation (8.4) takes the form $U_T/V_T \sim U_0/V_K^* \sim U_0/V_K(V_T/V_T^*)^{3/4}$. Estimating $V_T^*/V_T \sim U_T/V_T$, we obtain

$$U_T/V_T \sim (U_0/V_K)^{4/7} \tag{8.6}$$

The exponent 4/7 is shown on Figure 3 and is in good agreement with the BLL data.

Very fast flames

As the ratio U_0/V_K increases, so does the ratio R_0/R_K and the transitional region becomes larger in comparison with the inertial interval. The burning velocity starts to deviate from the asymptote specified by Eq.(8.1) towards $\gamma_2=1$. This is more noticeable for the flows with the lower Re numbers. Asymptote 3 in Table III seems to be intermediate and it can not be reliably determined form the BLL data.

If the intensity of turbulence V_T becomes negligibly small in comparison with U_0, one can expect that $U_T = U_0$. The function $U_T = U_0$ corresponds to the line $Y = X$ shown on Figure 4. It is clear that, in the range of the BLL data, this asymptote is not fully achieved.

9. SLOW FLAMES

These flames are characterized by $K > 1$ and relatively small ratios U/V. The asymptote in Table III specifies the burning velocity as

$$\frac{U_T}{V_T} = \gamma_3 \left(\frac{U_0}{V_K}\right)^{\gamma_4} \left(\frac{R_T}{R_K}\right)^{\gamma_5} \mathrm{Le}^{\gamma_6} \tag{9.1}$$

$$\gamma_3 \approx 6 \quad \gamma_4 \approx 1, \ \gamma_5 \approx -1/4, \ \gamma_6 = -(\gamma_4+3\gamma_5)/2 \approx -1/8$$

The regimes specified by Eq.(8.1) and Eq.(9.1) are separated at $K \sim 1$ by the intermediate zone where both of the asymptotes are not valid. We assume below that $\mathrm{Le} = 1$.

For slow flames, the functions $\overset{\circ}{F}_K(a)$ and $F_T(a)$ are the linear functions which differ only by a constant so that $F_I(a) = F_T(a) \approx \overset{\circ}{F}_K(a)+\gamma_7$, where

$\overset{\circ}{F}_K(a) \approx 4a$ and $\gamma_7 \approx 7$. These equations can be determined from Tables I and II by assuming that $a \to -\infty$. The inertial interval function $F_I(a)$ is similar to $F_T(a)$. The derivatives of the scaling functions are given by $F'_I = F'_T = F'_K \approx 4$. The value D^* determined by Eq.(6.5) is constant and the fractal dimension of the slow flames

$$D \approx 2\frac{1}{12} \qquad (9.2)$$

is only slightly greater than 2. Equation (6.3) can be written as $U_T/U_0 = C_U(R_T/R_0)^{D-2}$ where C_U is a constant factor. This factor is normally neglected in the fractal analysis since the term R_T/R_0 is most significant. If $D = 2$ then it is the factor C_U which determines U_T/U_0.

In most simple case, the chemical reactions can be characterized by one parameter - the characteristic reaction time $\tau_0 \equiv \delta_0/U_0 = \nu/U_0^2$. Using this parameter we can rewrite Eq.(9.1)

$$U_T = 8.2\, U_0 \mathrm{Re}^{\gamma_8} = 8.2\left(\frac{\nu}{\tau_0}\right)^{1/2} \mathrm{Re}^{\gamma_8}; \qquad \gamma_8 \approx 1/16 \qquad (9.3)$$

Williams (1985) suggested similar approximation with $\gamma_8 = 0$. The value of γ_8 determined from the BLL data is quite small and, if we take into account scattering of the original experimental data, it is not clearly distinguishable from zero. This point is illustrated on Figure 4 where the line $Y \sim X$ corresponds to $U_T \sim U_0$. Equation (9.3) specifies the flame whose propagation speed explicitly depends on the viscosity coefficient ν. Thus, the flame must have some zones comparable with the viscous scales. The flame is strongly influenced by the smallest fluctuations ($U \sim 6U_0$) but the turbulent propagation velocity U remains almost constant inside the inertial interval. Due to the influence of the fluctuations of the smallest scales the turbulent propagation speed is essentially greater than the laminar propagation speed: $U_T \sim 8.2\, U_0$ in Eq.(9.3). Within the inertial interval, the fractal dimension D is only slightly greater than 2. The flame is not dynamically coupled with turbulence and, probably, can be treated as a passively convected interface. As it can be seen on Figure 2, the theoretical approximation of the function S, which is based on the consideration of the premixed flames as flamelets, is well-matched by the function S_I determined from the BLL data.

Although the wrinkled laminar flames can not exist in the case of large K (see Williams, 1985; Pope, 1987; Bradley, 1992), the flamelets may exist in the regions of relatively small local dissipation rate where the local value of K is not so large. The dissipation rate has quite significant fluctuations and the regions of relatively small dissipation rate and relatively small velocity gradients coexist with the regions where the dissipation rate is much greater. Kuznetsov and Sabelnikov (1989) and Bradley et al. (1992) put forward arguments that the flamelets are likely to exist in the regions with the low local dissipation rate. In the rest of the flow the flame front is stagnant or locally quenched. The reactions in the flamelets are most intense and determine the propagation velocity. This explains the strong dependence of U_T on ν in Eq.(9.3) as well as the weak dependence of U_T on Re in $U_T \sim U_0 Re^{\gamma 8}$. The flame front is fragmented and does not form a continuous surface. The large-scale velocity fluctuations can not effectively stretch the flame front. Its fractal dimension within the inertial interval remains close to $D = 2$ (which is the fractal dimension of a plane). The most of flame wrinkling occurs at smallest scales and this wrinkling significantly accelerates the flame.

10.CONCLUSIONS

1) A new mathematical formulation of the cascade hypothesis for premixed combustion is suggested. The new formulation is consistent with the premixed combustion theories by Yakhot (1988) and Sivashinsky (1988) which are based on the cascade hypothesis. The new formulation has more degrees of freedom in choosing the scaling functions since the present consideration involves fewer modeling assumptions. Basically, we postulate only the cascade hypothesis and require satisfaction of certain consistency principles. The turbulent burning velocity dependence on the main parameters is determined by the two one-variable scaling functions $F_K(a)$ and $F_T(a)$. Unlike in previous investigations, the scaling functions are determined directly from experimental data rather than from theoretical models.

2) The cascade hypothesis imposes certain restrictions on the dependence of the turbulent burning velocity on the Karlovits stretch factor K and Reynolds number Re. The correlations of the burning velocity data by Bradley et al. (1992) (BLL data) which comprise most of the known measurements of the turbulent burning velocity are shown to satisfy the these restrictions. This confirms the cascade hypothesis and validates its use for premixed combustion.

3) Two clear asymptotic regimes - fast flames and slow flames - have been detected. (The fast flames asymptote has been pointed out by Bradley et al. (1992)). The fast flames correspond to the wrinkled laminar flame regime and

they are separated from the slow flames by $K \sim 1$.

4) It is shown that, for the fast flames, the undisturbed laminar flames and the flames which have been wrinkled by the fluctuations of smaller scales (fully-developed flames) are affected by the larger turbulent fluctuations in different ways. In terms of the scaling functions, this means that the functions $F_K(a)$ and $F_T(a)$ are different.

5) Sivashinsky (1988) introduced the scaling function $S(V/U)$ and modeled this function for constant density turbulence. The function S specifies the increase the flame propagation velocity (whose initial value is $U(R)$) by the turbulent fluctuations of intensity V and scale R. The function $S_I(V/U)$ is a similar function determined from the BLL data. The agreement of the scaling functions S and S_I is good for $V/U > 0.5$. In the region $V/U < 0.5$, the BLL data indicate more rapid increase of the propagation velocity than it is predicted by the function S.

6) On the basis of the cascade hypothesis, various flame characteristics are determined from the BLL data. It is shown that the fractal dimension of the fast flames is $D = 7/3$. This confirms, for the fast flames, Krestin's (1989) hypothesis of dynamic coupling between the flame fronts and turbulence.

7) It is shown that the ratios of the effective inner cutoff R_0 and the Kolmogorov length scale R_K, which are determined from BLL data for the fast flames, are larger for larger ratios of U_0/V_K. This is correctly predicted by Sivashinsky's function S and by Peters's (1986) concept of the Gibson scale. These theories, however, overestimate the ratio R_0/R_K.

8) According to the scaling functions determined from the BLL data, the fast flames (with $V/U \to 0$) are wrinkled by the turbulent fluctuations more efficiently than passively convected interfaces. This can be explained by the influence of the flame generated turbulence. The consideration of these factors yields the 4/7 exponent which, as it is shown on Figure 3, is close to the exponent 0.6 determined from the BLL data.

9) The asymptote determined here from the BLL data for the slow flames is quite close to the approximation of the turbulent burning velocity suggested by Williams (1985). It is likely that this regime corresponds to fragmented flamelets.

ACKNOWLEDGMENTS

The author thanks Dr. P.A.Jakobs for useful comments.

Bradley, D. (1992). How fast can we burn?. *XXIV Symposium (International) on Combustion*. The Combustion Institute, Pittsburgh, p.247.

Bradley, D., Lau, A.K.C. and Lawes, M. (1992). Flame stretch rate as a determinant of turbulent burning velocity. *Phil. Trans. R. Soc. Lond. A,* **338**, 359.

Clavin, P. (1985) Dynamic behavior of premixed flame fronts in laminar and turbulent flows. *Prog.Energy Combust. Sci.* **11**, 1.

Gouldin, F.C. (1987). An application of fractals to modeling premixed turbulent flames. *Comb.Flame* **68**, 249.

Krestin, A.R. (1988). Fractal dimension of Turbulent Premixed Flames. *Comb. Sci. and Tech.* **60**, 441.

Landau, L.D. (1994). On the theory of slow combustion. *Acta Phisicochim.* USSR, **19**, 77.

Klimenko, A.Y., Bilger, R.W. and Roomina, M.R. (1995). Some PDF integrals for self-similar turbulent flows. *Combust. Sci. Tech.* **107**, 403.

Kuznetsov, V.R. and Sabelnikov, V.A. (1989). *Turbulence and Combustion*. Hemisphere.

Monin, A.S. and Yaglom, A.M. (1975). *Statistical Fluid Mechanics*. The MIT Press, Vol.2.

Peters, N. (1986). Laminar flamelet concepts in turbulent combustion. *XXI Symposium (International) on Combustion*. The Combustion Institute, Pittsburgh, p.1231.

Pocheau, A. (1994). Scale invariance in turbulent front propagation. *Phys. Rev. E* **49**, 1109.

Pope, S.B. (1987). Turbulent premixed flames. *Ann. Rev. Fluid Mech.* **19**, 237.

Sivashinsky, G.I. (1988). Cascade-renormalization theory of turbulent flame speed. *Combust. Sci. Tech.* **62,** 77.

Williams, F.A. (1985). *Combustion theory*. Addison-Wesley.

Yakhot, V (1988). Propagation velocity.of premixed turbulent flame. *Combust. Sci.Tech.* **60,** 191.

Zeldovich, Ya.B. (1982). Exact solution of the problem of diffusion in a periodic velocity field. *Soviet Physics, Doklady* **27** (10), 797.

TABLE I. Representation of the scaling functions

Function	Representation	
	in the region $\overset{\circ}{a}_K \le h_a$	in the region $\overset{\circ}{a}_K \ge h_a$
$\overset{\circ}{F}_K(a)$	$-F_h(a, \mathbf{h}_K^-)$	$F_h(a, \mathbf{h}_K^+)$
$F_T(a)$	$-F_h(a, \mathbf{h}_T^-)$	$F_h(a, \mathbf{h}_T^+)$
$F_h(a, \mathbf{h})$	$h_1 + h_2 a + \exp(h_3 a + h_4)$	

TABLE II. The values of the constants

j	1	2	3	4
$(h_K^-)_j$		3.91	4.45	4.23
$(h_T^-)_j$	$(h_K^-)_1$-7.04	3.95	7.3	2.29
$(h_K^+)_j$	$(h_K^-)_1$-59301	-11.94	-13.03	28.55
$(h_T^+)_j$	$(h_K^+)_1$+5.19	-17.05	-21.37	34.23
h_a	1.45			

TABLE III. Main asymptotic regimes

1. Slow flames:	$a_T = 0.99\overset{\circ}{a}_K - 0.25\overset{\circ}{b} + 1.78$
2. Fast flames:	$a_T = 0.61\overset{\circ}{a}_K + 0.27$
3. Very fast flames:	$a_T = 0.70\overset{\circ}{a}_K - 0.06\overset{\circ}{b} + 0.3$

TABLE CAPTURES

TABLE I. Representation of the scaling functions

TABLE II. The values of the constants

TABLE III. Main asymptotic regimes

FIGURE CAPTURES

FIGURE 1. The scaling functions determined from the BLL data;

——— $1/F'_K(a)$;

- - - $1/F'_T(a) \approx 1/F'_I(a) = D^* - \frac{7}{3}$.

FIGURE 2. The scaling function $S\left(\frac{V}{U}\right)$;

——— S_I, determined from the BLL data;

- - - S, modeled by Sivashinsky (1988), $\beta = 1.4$.

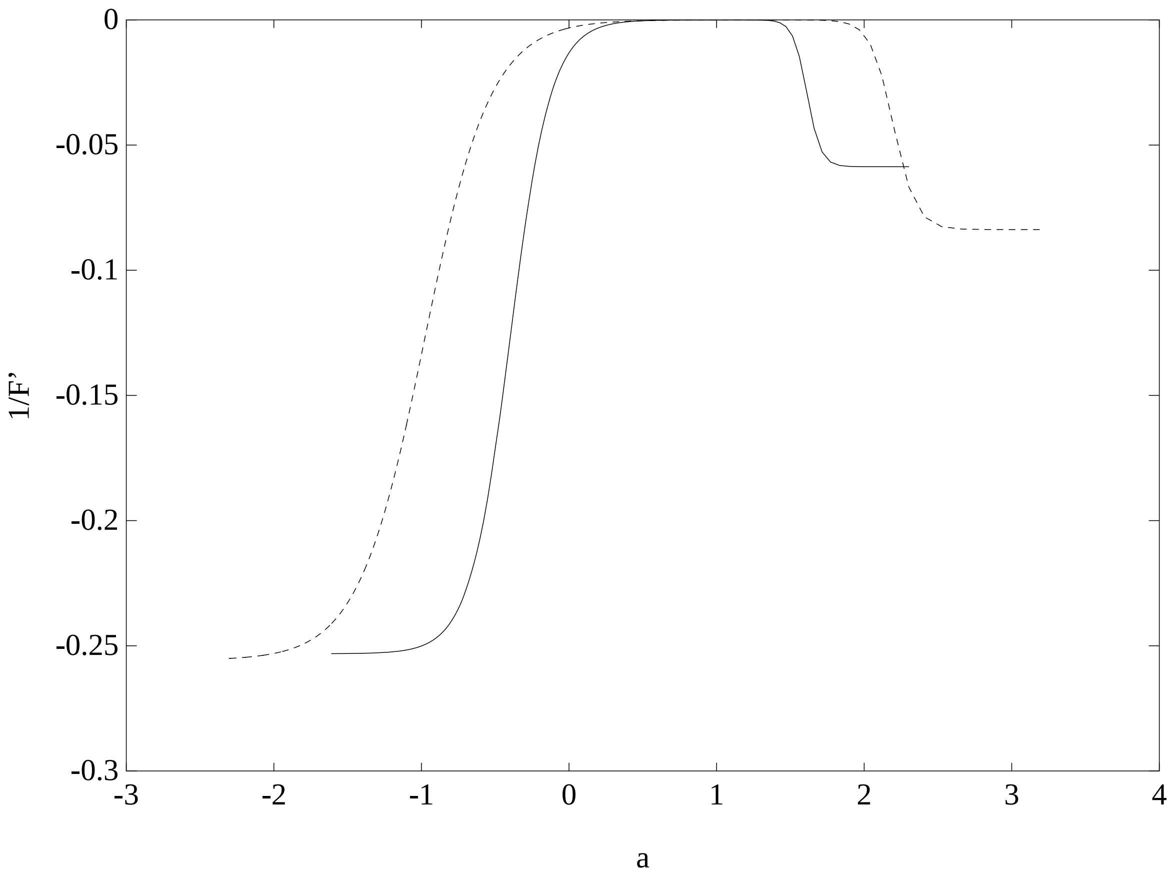
FIGURE 3. The dependence of $Y = U_T/V_T$ on $X = \mathrm{Le}^{-1/2}(U_0/V_K)$. Different curves correspond to $\mathrm{Re}/\mathrm{Le}^2 = \{$ 10, 20, 50, 100, 250, 500, 1000, 2000, 3000, 4000, 5000, 6000 $\}$. The arrows show the direction of $\mathrm{Re}/\mathrm{Le}^2$ increase.

——— BLL data

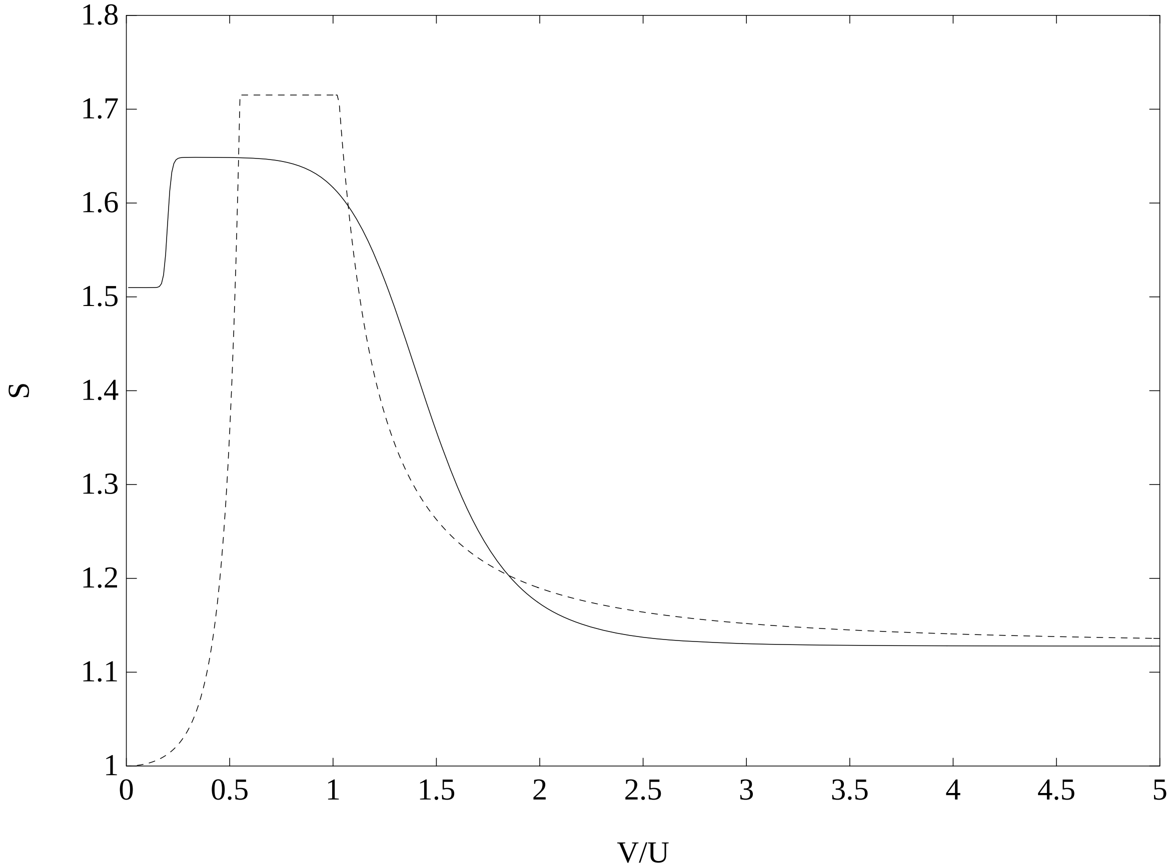
- - - approximation based on the cascade hypothesis (shifted, $Y = (U_T/V_T)/3$)

········ The line $K\mathrm{Le} = 1$ is shown.

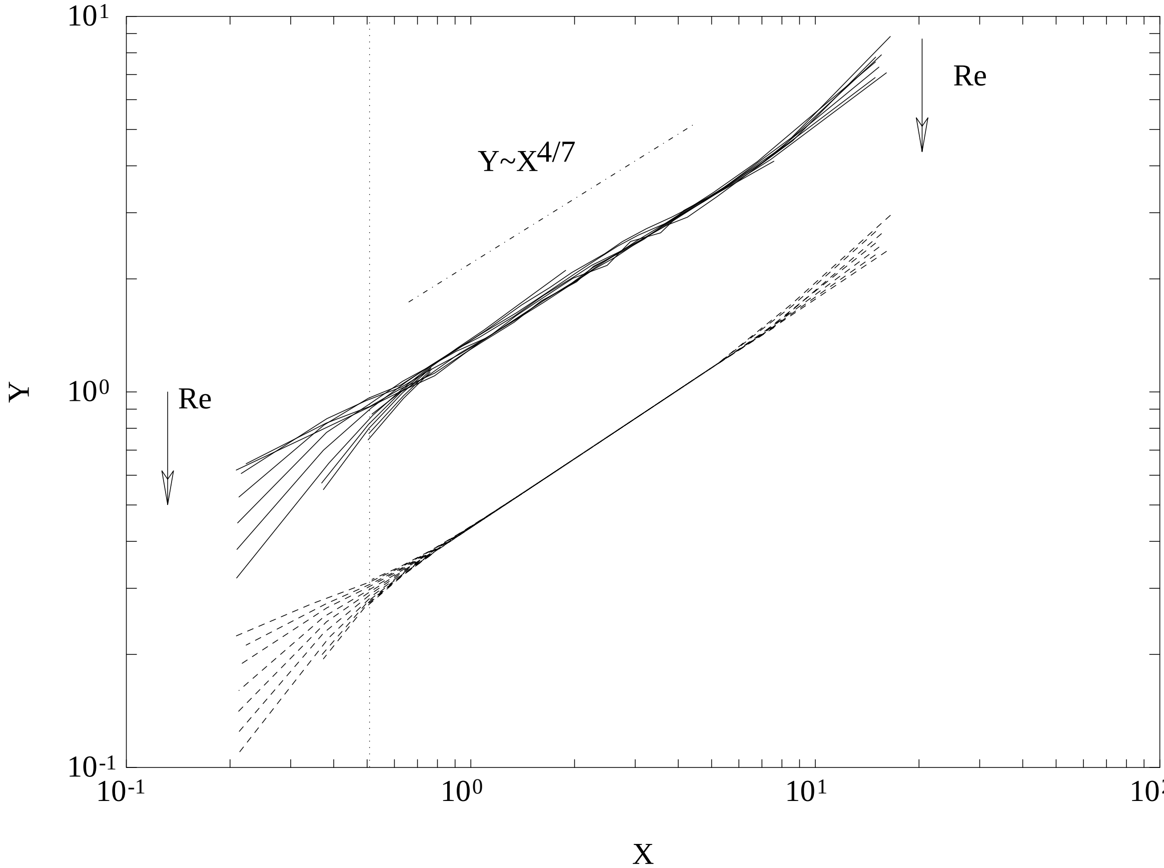
FIGURE 4. The dependence of $Y = U_T/V_T$ on $X = U_0/V_T$. See Figure 3 for key.



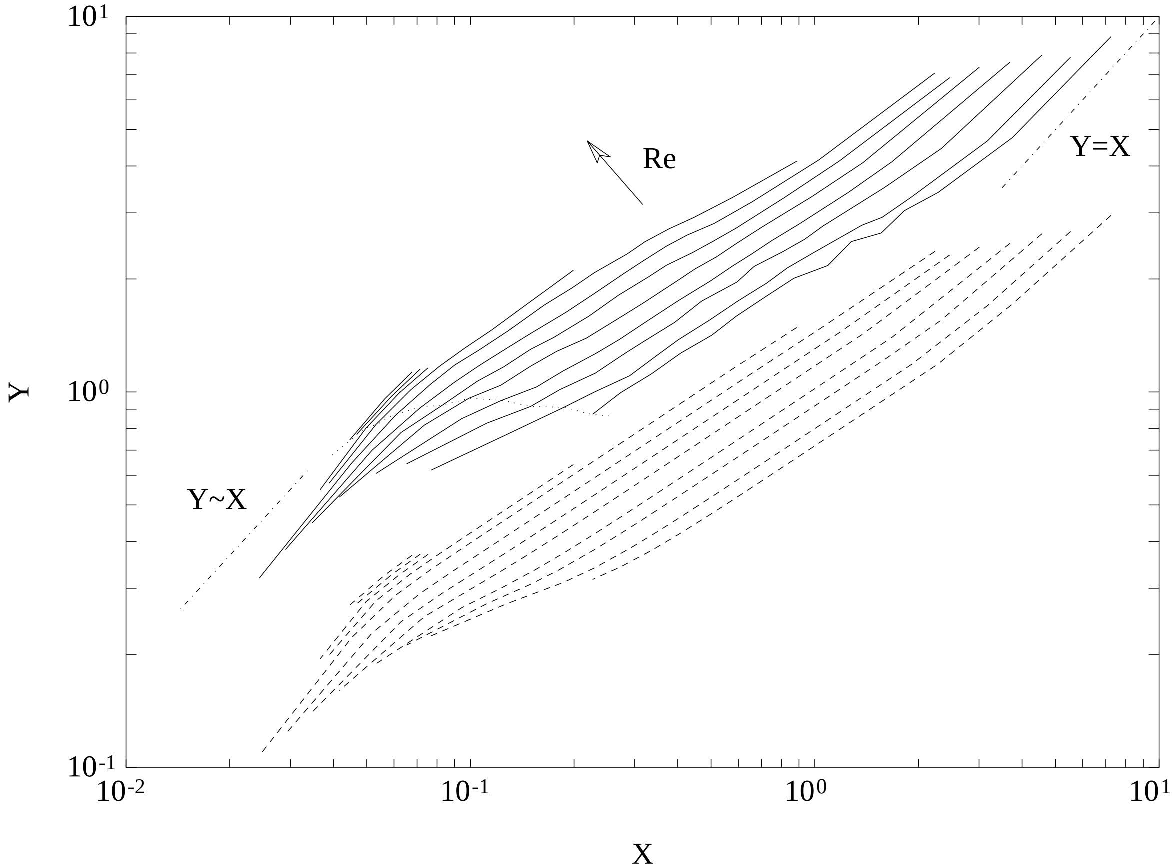
1/F'
a
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-3
-2
-1
0
1
2
3
4



S
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
V/U



Y~X^4/7
Re
Re
Y
X
10^1
10^0
10^-1
10^-1
10^0
10^1
10^2



Re
Y=X
Y~X
Y
X
10^1
10^0
10^-1
10^-2
10^-1
10^0
10^1