Computer Simulations of Abstract Competition

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ABSTRACT

This paper investigates major properties of abstract competition, which deals with competition in its most generic form and is not necessarily directly related to competition that can be observed in biological, economic or other realistic systems. Generic properties of competing systems are investigated theoretically and computationally and major results are summarized in the present paper.

INTRODUCTION

It would be difficult to determine the instance when the idea of existence of similarities between complex competing systems of different physical nature was first mentioned in the literature – similar ideas were repeatedly advanced by prominent representatives of very different fields of knowledge. Czech philosopher Radovan Richta [1] coined the term technological evolution referring to technological progress resulting in replacement of manual labor by machines. British theoretical physicist Stephen Hawking [2] related the progress of humanity to replacing genetic forms of information by social forms. It seems, however, that it was British evolutionary biologist Richard Dawkins [3] who put forward the ideas of similarity between biological and social forms of evolution in most clear and explicit terms. He suggested that information unites called memes evolve in human society in a manner similar to evolution of genes in biological systems. This idea gave origin to a field called memetics and pointed out that large competing systems evolve not through gradual progress but in a cyclic manner. These cycles are known under different names in different fields of science but, in their generic form, they can be called leaping cycles. Typically, a leaping cycle involves rapid expansion, stable domination and sudden collapse. Progress in a leaping cycle can be visualized by a frog moving forward in a series of leaps (this analogy is incomplete as progress can also be reversed as a part of the cycle). This term has links with “Great Leap Forward” introduced by Dimond [8] to characterize rapid progress of human race 50000 years ago.

The distinctive purpose of the present work is investigation of competition in its most abstract form. Abstract competition is not intended to model any
specific biological or economic system and does not simulate any specific historic or evolutionary event but it investigates competition properties generically, at a conceptual level. The computer simulations presented in this work are generic and do not necessarily have a specific prototype process in the real world. The concept of abstract competition differs substantially from evolutionary programming [4] whose purpose is in using evolutionary principles to build efficient algorithms solving specific problems which may have nothing to do with evolution.

ABSTRACT COMPETITION

Competition in its generic form involves competing elements. These elements should have some autonomy and posses a set of properties determining their competitiveness. The process of competition is conducted according to competition rules determining winners and losers but may also involve some degree of randomness. The elements can move in physical space and this motion can have both deterministic and random components.

It can be seen that competing elements as we defined them are very similar to the concept of stochastic particles introduced by Pope [9] and expensively used in modeling of reacting flows (see the more recent reviews [10,11] of related stochastic modeling methods and analysis [12] of stochastic particle properties for further details). Stochastic particles move in physical space, possess a set of properties, which are modified by reactions and mixing. The essential difference between competing elements and conventional particles comes from the mixing operation: conventional mixing is dissipative and conservative while competition is not. We still call competing elements “particles”, implying that we are dealing with generic elements possessing a set of properties but introduce competitive mixing to reflect the competitive character of the simulations.

COMPETITIVE MIXING

Among many important properties of conventional mixing [9], we focus on three of them. The conventional mixing is

- Conservative – preserving the sum of particle properties
- Dissipative and non-segregating -- reducing differences between the properties
- Non-discriminating -- particles have symmetric contributions to the mixing outcomes

The main feature of competitive mixing is its discriminating character: there are winners and there are losers. Two types of competitive mixing can be considered

- Resource competition: conservative and segregating competition for a limited resource resulting in redistribution of a fixed total resource amount in favor of the competition winners.
- Information competition: non-segregating and non-conservative competition resulting in replicating information carried by the winners.

In light of the issues discussed in introduction, it seems that the information competition is most interesting and this topic is studied here. Any type of mixing can be

- Two-particle mixing --- each elementary mixing event involves only two particles
- Multi-particle mixing --- each elementary mixing event involves many (more than two) particles

Both two- and multi-particle mixing are commonly used in conventional modeling of mixing [9-12]. Here, we note the extreme complexity of competitive multi-particle mixing and restrict our consideration only to two-particle mixing. The terms “competitive mixing” and “competition” can be used interchangeably, although the first term places the emphasis on redistribution of particles properties resulting from the competition.

TRANSITIVENESS OF THE COMPETITION

During competition particles form competing groups (i.e. couples for two-particle competition) and for each couple a winner and a loser are determined by the competition rules. If A is a winner in competition with B, we will denote this as A>B. In a pure competition B then loses its properties and is assigned the properties of A (in principle, B may retain a fraction of its old properties but this case is not considered in the present work). We call competition transitive if

A>B and B>C means that A>C

For any A, B and C. Competition is intransitive if a triplet where

A>B and B>C but A<C

exists .

Transitive competition can be practically represented by a numerical ranking function:

A>B when and only when rank(A)>rank(B)

while intransitive competition can not. This does not mean that a reasonable ranking function can not be introduced for intransitive competition but, if competition is intransitive, ranking does not determine the competition outcomes. That is if rank(A)>rank(B) then A>B is more likely (i.e. is valid for the majority of A and B) but A<B is also possible. The case of
rank(A)>rank(B) and A<B can be called competition error.

The simplest example of intransitive competition is given by the scissors-paper-rock game where paper wins over the rock but loses to scissors while scissors lose to the rock. Practically, intransitivity is linked to multiplicity of competition criteria. For example, in the 100m run competition, the rule is very simple: the fastest runner wins, while the competition of two tennis players is more complicated. Being faster, taller, having better service or better return – all these multiple parameters characterizing a tennis player can contribute to a good performance but neither of them can guaranty a victory, especially if the competing players have close ranks. Intransitive cases when A>B>C>A are common in tennis. Real life competition is even more complicated --- the criteria distinguishing winners and losers are numerous and fuzzy. Intransitivity (more accurately, a complex combination of transitive and intransitive rules) is most common in the real world.

MUTATIONS

Existence of mutations is a common property of evolving competing systems and, thus, it is essential feature of abstract competition. The properties of winners remain without change, but the process of transferring information from winners to losers is subject to mutations. The following features of the mutations must can be stressed:

- Mutations are random and do not have any purpose or direction
- Mutations are more likely to be detrimental than useful, especially when the properties approach the point of high competitiveness

We can distinguish background mutations, which are common low-intensity mutations, and rare mutations, which may generate a significant change. Both type of mutations are essential for evolution although in real systems the distribution of mutation strength is more continuous and rare mutations are represented by the tail of the distribution. In the present simulations we use a simpler model introducing mutations of three categories of increasing strength and decreasing frequency: background, reformatory and revolutionary mutations. It should be noted that these mutations are completely random and do not have any purpose or goal. Overwhelming majority of mutations representing reforms and, even more so, revolutions are unsuccessful and disappear without a trace. The term “reformatory mutations” is used to emphasize that these mutations may modify existing structures but are insufficiently strong to cause a more significant upheaval.

LOCAL AND GLOBAL COMPETITIONS

Particles representing competing units in abstract competition can move in physical space. This motion is presumed to have deterministic and random components. In the present work, particles moves according to the Ito stochastic differential equations and independently of their properties, but in general the influence of stochastic jumps and motion interactions with particle properties may be also of interest. Competitive mixing can be

- Local, when only particles that are closest in physical space are allowed to form mixing groups
- Global, when locality restrictions do not apply and distant particles may form a mixing group

Note that the effect of stochastic jumps would be in introducing global effects into localized competition. Here, we restrict our consideration to particle motions represented by Markov diffusion processes.

In the real world, competition is usually localized by obvious physical constraints as competitors have specific physical locations in most cases. Competition between local shops can be seen as example of local competition while competition between internet sellers is global.

COMPETITIVE SIMULATIONS

Competitive simulations are performed here in two-dimensional physical space. Particle motions are represented by two-dimensional Ornstein–Uhlenbeck stochastic process resulting in Gaussian distribution of particles in the physical space. The images of physical space are presented as square domains by mapping the unbounded physical space on a unit square using “erf” functions along each coordinate \(x_1\) and \(x_2\). The presented simulation involve up to 64000 particles.

The particle properties are represented by three values \(Y_1, Y_2\) and \(Y_3\). The competition rule is given by

\[A > B \text{ if } \sum_{i} \text{sign}(Y_{iA} - Y_{iB}) < 0\]

This means that A wins over B if A wins over B in at least two out of three categories, where A wins over B in category \(i\) if \(Y_{iA} < Y_{iB}\). The case of \(Y_{iA} = Y_{iB}\) is statistically insignificant and can be treated differently with practically the same outcomes.

The particle properties are subject to the following resource constraint

\[Y_1 + Y_2 + Y_3 = 1\]

With this constraint, the property domain becomes a two-dimensional equilateral triangle. The strongest property combinations are located at the vertices of a triangle and the weakest is at the triangle center. The particle strength
can be assessed by the ranking function rank(A) which simply represents the distance from the triangle center.

It is easy to see that competition considered here is intransitive, and even more so locally intransitive (i.e. intransitive triplets can be found in vicinity of every point). This example represents the simplest locally intransitive competition and for this simplicity allows for complete representation of the state of the system by color distribution in the physical space. In intransitive competition considered above, rank(A) does not determine the competition winner. We also consider a transitive version of this competition where rank(A) is the sole criterion for the competition outcome: A>B when and only when \( \text{rank}(A) > \text{rank}(B) \).

**COMPETITIVE ESCALATION AND COMPETITIVE DEGRADATION**

Diffusion approximation is conventionally applicable to describe long-time asymptotics of a process representing a superposition of a large number of small random steps. This approximation can be inaccurate to describe the competition process even if only small background fluctuations are present. The evolution of a competing system is largely determined by the leading group – the group of particles with highest ranking. Competition may result in two major outcomes:

- **Competitive escalation** – continuing increase of ranking of the leading group and the rest of the particles
- **Competitive degradations** – slow but persistent decrease in ranking of the leading group and other particles

Note that competitive degradation is impossible in transitive competition – the leading particle (i.e. particle with the highest rank) can not lose competition to any other particle and can not be moved in the property space as long as it remains the leading particle. The leading particle, however, can be overtaken by another particle due to mutations but then the overtaking particle become the leader and the rank of the leader increases – this corresponds to competitive escalation, not degradation.

In the present work we obtain expressions for the rates of competitive degradation and competitive escalation in terms of the parameters of the competition and determine that degradation can dominate over escalation when competition is local and intransitive. Under these conditions, particles tend to form cooperative pyramidal structures visible in Figure 1 which shows rapid expansion of strong red structure into chaotic domain populated by short-living formations. These structures can be quite effective in exercising control over the whole domain or a part of it. We use the term “cooperative” to denote the case when the level of structural competition is decreased – the competing particles have similar properties and tend to be subject to relatively small property adjustments. Fierce competition of particles with radically different properties tends to occur at the boundaries separating different structures. Intransitiveness and localness of the competition allows for a degree of tolerance between intra-structural particles but, over a long period of time, inevitably results in degradation. Competitive degradation is accumulation of a large number of competition errors: each of them is not noticeable but these errors tend to accumulate into overall reduction of ranking.

Transitive or global competitions result in competitive escalation but do not form structures and competition remains plain — this can be metaphorically expressed as “domination of crocodiles” (i.e. domination of relatively simple but effective competitors that may last for a long time without any substantial development). Examples of competitive escalations and degradations are numerous in the real world. An arms race is a competitive escalation, while aging and bureaucratization represent competitive degradations.

Can the process of competitive degradation be halted? It seems that the answer is negative a long run (at least without changing the nature of the system). Although competitive degradation can not be eradicated, it can be delayed or remedied through reformatory mutations. These mutations may bring in a new, more effective leadership. The system then has to go through some adjustment increasing its ranking; however the degradation does not disappear and will resume again as soon as reforms are finished. If reformatory mutations do not introduce good leadership, the structure continues to slide down in ranking up until the system collapse becomes inevitable. Practical examples of this consideration can be drawn from political science: democratic elections of a new government allow for invigorating the political system through relatively minor adjustments. A state with a dictatorial ruler (who may be quite effective in a short run) will tend to degrade to the point, where either a radical political upheaval or complete collapse becomes inevitable.

**THE LEAPING CYCLE**

Cyclic behavior can be observed in many competing systems. Business cycles are widely known and proved to be quite resilient --- they have not been eradicated (with exception of extreme cases of joint eradication of cycles together with the economy itself) in spite of continuing efforts of governments and central banks. It is also widely known that evolution of life on Earth has passed through periodic extinctions. Historic cycles are less known, although a number of authors (some of them may be viewed as controversial but this is not related to historical cycles), for example Le Bon [13] and later Gumilev [14], detected cycles in development of nations.
Technology is also known to progress in cycles of successive technological generations. Even science, as discovered by Kuhn [15], is not free of cycles of successive paradigms.

The hypothesis, suggested in ref.[7], proposes that all these cycles have common features pertaining to generic properties of competition. The common denominator, called the leaping cycle, is characterized by a dynamic growth in a new direction, stable dominance with slow weakening, which initially can hardly be noticed, and then by a rapid collapse. This hypothesis is tested here: if the leaping cycle is a common feature of complex competitive systems then it can be expected to appear in computer simulations of abstract competition. The cycle has indeed appeared in abstract simulations based on localized intransitive competition. It should be noted that the simulations demonstrate certain features of competitive evolution but not the evolution itself: the complexity of the problem under consideration is restricted to a two-dimensional domain and can not increase with time.

Figure 2 displays a long time series of mean and maximal values for $Y_1$, $Y_2$ and $Y_3$. Graphically, each of these values is associated with a primary color, red, green or blue. The maximal values tend to increase due to rare successful mutations and gradually decrease due to competitive degradation. Periods of dominance of different primary colors are clearly visible in the figure. A successful mutation may result in rapid expansion of a color followed by a period of stable domination. The maximal value (determining rank of the leading particle) tends to slowly weaken with time. Initially, this weakening process can hardly be noticed – in fact color expansion may still occur at this stage. However, as the strength of the leading group decreases, its dominance is increasingly challenged. The early challenges are usually fended off by the dominant structure. Sometimes, a successful mutation boosts rankings of the leading structure, but degradation always resumes. The fate of weakening structure is determined by luck: a very strong successful mutation on the opposite side may annihilate the structure prematurely. If no strong opponents are encountered, the structure weakens further and the situation becomes chaotic, with multiple colors competing and replacing each other at a fast pace. The chaos continues up until a new strong structure leaps into stable dominance.

**CONCLUDING REMARKS**

Abstract competition studies properties of competition in most generic form, which is not intended to be a model of any specific competition process. Competition which is both intransitive and local is characterized by forming cooperative hierarchical structures with reduced level of intra-structural competition. These structures can exercise effective control over computational domain but are subject to the process of competitive degradation resulting in slow weakening of the structures. Simulations over long periods of time indicate cyclic behavior: successful mutations may result in rapid expansion, followed by stable domination (gradually eroded by degradation) and rapid collapse.


Figure 1. State of the simulations after 85400 steps: triangular property domain (left) and physical domain (right).

Figure 2. Long time series for intransitive localized simulations involving around 64000 particles; notations: —— average values, - - - -, maximal (leading) values, the vertical line corresponds to the state shown in Figure 1.