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# Matching the conditional variance as a criterion for selecting parameters in the simplest multiple mapping conditioning models

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The simplest model within the multiple mapping conditioning (MMC) approach, that involves a single mixture-fraction-like reference variable, is considered in the Brief Communication. An important parameter—the minor dissipation time—remains unknown in the probabilistic version of the model. The present work demonstrates by the specially developed asymptotic analysis that the simplest MMC possesses an ability (although somewhat limited) to match the physical intensity of the conditional fluctuations and this match represents the criterion for proper selection of the minor dissipation time. © *2004 American Institute of Physics*. [DOI: 10.1063/1.1803742]

The multiple mapping conditioning (MMC) approach[1] to turbulent nonpremixed combustion is characterized by dividing all fluctuations of the reactive species (i.e., the composition space accessible by the species[2]) into major and minor. The major fluctuations are treated with assistance of the reference variables while the minor fluctuations are either neglected (conditional MMC) or treated by conventional mixing models (probabilistic MMC).[1,3] In its treatment of major fluctuations, the MMC approach is compliant with all mixing criteria (such as linearity, independence, localness, boundness, etc.[1,3] Effectively, MMC unites the conditional methods [conditional moment closure[4] (CMC)] with the probability distribution function (PDF) methods [mapping closure, interactions by exchange with the mean (IEM) or conditional mean (IECM), Curl's and other models reviewed in Refs. 5 and 6]. The MMC approach can be formulated by using the following stochastic Ito equations:

$$d\mathbf{x}^* = \mathbf{U}(\boldsymbol{\xi}^*,\mathbf{x}^*,t)dt, \tag{1}$$

$$d\xi_k^* = A_k^\circ(\boldsymbol{\xi}^*,\mathbf{x}^*,t)dt + b_{kl}(\boldsymbol{\xi}^*,\mathbf{x}^*,t)dw_l^*, \tag{2}$$

$$dZ_I^* = (W_I^* + S_I^*)dt, \tag{3}$$

where

$$S_I^* = \frac{\overline{Z_I^*} - Z_I^*}{\tau_S}, \quad \overline{Z_I}(\xi,\mathbf{x},t) \equiv \langle Z_I^* | \boldsymbol{\xi},\mathbf{x}\rangle. \tag{4}$$

$x^*$ traces motions in the physical space, $\mathbf{Z}^* = \{Z_1^*,\ldots,Z_{n_s}^*\}$ models the reactive species, $\boldsymbol{\xi}^* = \{\xi_1^*,\ldots,\xi_{n_r}^*\}$ represents the stochastic reference variables, and $w_l^*$ is the standard vector Wiener process. The asterisk superscript is used to outline the stochastic nature of the variables while the abbreviated form is used for conditional averages. For example, $\overline{Z_I} = \langle Z_I^* | \boldsymbol{\xi},\mathbf{x}\rangle$ denotes $\langle Z_I^* | \boldsymbol{\xi}^* = \boldsymbol{\xi}, \mathbf{x}^* = \mathbf{x}\rangle$ and $\overline{Z_I^*} \equiv \overline{Z_I}(\boldsymbol{\xi}^*,\mathbf{x},t)$. The small indices $i$, $j$, $k$, $l=1,\ldots,n_r$ run over all reference variables and capital indices $I$, $J=1,\ldots,n_s$ run over all reacting species while $n_r$ is selected so that $n_r \leqslant n_s$.

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In conditional MMC, $\overline{Z_I^*}$ is interpreted as a model for reactive scalars while, in probabilistic MMC, $Z_I^*$ is treated as the reactive scalar value. The role of the operator $S_I^* = S[Z_I^*]$, which can be called the "minor dissipation operator," is to keep $Z_I^*$ close to $\overline{Z_I^*}$ and the probabilistic and conditional versions of MMC are reasonably similar but not identical. Determining a proper value of the "minor dissipation time" $\tau_S$ (i.e., the characteristic dissipation time of the operator $S$) is the focus of the present work. In conditional MMC, the deviations of $\mathbf{Z}^*$ from $\overline{\mathbf{Z}}^*$ are considered to represent the numerical noise and the operator $S$ suggested in the diffusing clouds method[1] is optimized for determining $\overline{\mathbf{Z}}$. In probabilistic MMC, the minor dissipation time $\tau_S$ must be selected to match the physical properties of turbulent mixing. Although MMC can be combined with different forms of the operator $S$ representing conventional mixing models (for example, Curl's model, diffusing clouds, etc.), $S$ is always required to satisfy $\langle S_I^* | \boldsymbol{\xi},\mathbf{x}\rangle = 0$ and some other restrictions.[1] In most of the present work we use a simple linear relaxation (4) which is similar to the IEM (or IECM) model. The value $W_I^*$ represents the reactive source term which is assumed to be a function of the reactive scalars so that $W_I^* = W_I(\mathbf{Z}^*)$ in probabilistic MMC and $W_I^* = W_I(\overline{Z}^*)$ in conditional MMC. In this Brief Communication, we, generally, follow the former definition but introduce $\overline{W_I} \equiv \langle W_I^* | \boldsymbol{\xi},\mathbf{x}\rangle$.

The MMC model, formulated by the stochastic Ito equations (1)–(3) can be equivalently specified by the Fokker–Planck-type PDF equations.[1] All these equations have similar terms and, in order to avoid repetitions, we introduce the special drift/diffusion operators $\hat{D}^\circ$ and $\hat{D}$ and write the MMC equations in the form

$$\hat{D}^\circ(\underset{\mathbf{x}}{\mathbf{U}}, \underset{\xi}{\mathbf{A}^\circ}, \underset{\xi^2}{\mathbf{B}}, \mathbf{W} \underset{\mathbf{z}}{+} \mathbf{S})P(\mathbf{Z},\boldsymbol{\xi},\mathbf{x};t) = 0, \tag{5}$$

$$\frac{\partial \overline{\rho}}{\partial t} + \nabla \cdot (\mathbf{u}\overline{\rho}) = 0, \tag{6}$$

 

$$\hat{D}^{\circ}(\underset{\mathbf{x}}{\mathbf{U}},\underset{\boldsymbol{\xi}}{\mathbf{A}},-\underset{\boldsymbol{\xi}^2}{\mathbf{B}})\bar{\rho}P_{\xi}=0, \tag{7}$$

$$\hat{D}^{\circ}(\underset{\mathbf{x}}{\mathbf{u}_Z},\underset{\mathbf{Z}}{\mathbf{W}},-\underset{\mathbf{Z}^2}{\mathbf{N}})\bar{\rho}P_{Z}=0, \tag{8}$$

$$\hat{D}^{\circ}(\underset{\mathbf{x}}{\mathbf{U}},\underset{\bar{\mathbf{Z}}}{\overline{\mathbf{W}}},-\underset{\bar{\mathbf{Z}}^2}{\overline{\mathbf{N}}})\bar{\rho}P_{\bar{Z}}=0, \tag{9}$$

$$\hat{D}(\underset{\mathbf{x}}{\mathbf{U}},\underset{\boldsymbol{\xi}}{\mathbf{A}},\underset{\boldsymbol{\xi}^2}{\mathbf{B}})\bar{Z}_I=\bar{W}_I, \tag{10}$$

where

$$\langle S_I^*|\mathbf{Z},\mathbf{x}\rangle P_Z=\frac{\partial N_{IJ}P_Z}{\partial Z_J},\quad \bar{N}_{IJ}\equiv\langle N_{IJ}^{\circ}(\boldsymbol{\xi}^*,\mathbf{x}^*,t)|\mathbf{Z},\mathbf{x}\rangle, \tag{11}$$

$$N_{IJ}^{\circ}=B_{kl}\frac{\partial\bar{Z}_I}{\partial\xi_k}\frac{\partial\bar{Z}_J}{\partial\xi_l},\quad A_k^{\circ}\equiv A_k+\frac{2}{P_{\xi}}\frac{\partial B_{kl}P_{\xi}}{\partial\xi_l}, \tag{12}$$

$2B_{kl}=b_{ki}b_{li}$, $P(\ldots;t)$ is the joint PDF of its arguments at given $t$; the average density is modeled by $\bar{\rho}\equiv mP(\mathbf{x};t)$ with $m$ denoting the total mass in the domain under consideration; $P_Z(\mathbf{Z};\mathbf{x},t)\equiv P(\mathbf{Z},\mathbf{x};t)/P(\mathbf{x};t)$ and $P_{\xi}\equiv P(\boldsymbol{\xi},\mathbf{x};t)/P(\mathbf{x};t)$ are conditional PDFs; $P_{\bar{Z}}=P_{\bar{Z}}(\bar{\mathbf{Z}};\mathbf{x},t)$ is the PDF of the stochastic values $\bar{\mathbf{Z}}^*=\bar{\mathbf{Z}}(\boldsymbol{\xi}^*,\mathbf{x},t)$, which can be evaluated as $P_{\bar{Z}}=P_{\xi}\det(\partial\bar{\mathbf{Z}}/\partial\boldsymbol{\xi})^{-1}$; $\mathbf{u}\equiv\langle\mathbf{U}^*|\mathbf{x}\rangle$, $\mathbf{u}_Z\equiv\langle\mathbf{U}^*|\mathbf{Z},\mathbf{x}\rangle$, and $\mathbf{u}_{\bar{Z}}\equiv\langle\mathbf{U}^*|\bar{\mathbf{Z}},\mathbf{x}\rangle$ are the conditional velocities and $\mathbf{U}^*\equiv\mathbf{U}(\boldsymbol{\xi}^*,\mathbf{x}^*,t)$. The arguments of the operators $\hat{D}^{\circ}$ and $\hat{D}$ specify drift and diffusion coefficients respectively, in the spaces of the variables that are shown under the dotted line and applied to the functions that follow the operators. If a space indicator under the line is squared, then the value above the line represents a diffusion matrix while the other values represent drift vectors. Operator $\hat{D}^{\circ}$ is the conservative operator while $\hat{D}$ is the similar convective operator. For example, Eqs. (9) and (10) mean that

$$\frac{\partial\bar{\rho}P_{\bar{Z}}}{\partial t}+\nabla\cdot(\mathbf{U}\bar{\rho}P_{\bar{Z}})+\frac{\partial\bar{W}_I\bar{\rho}P_{\bar{Z}}}{\partial\bar{Z}_I}+\frac{\partial^2\bar{N}_{IJ}\bar{\rho}P_{\bar{Z}}}{\partial\bar{Z}_I\partial\bar{Z}_J}=0,$$

$$\frac{\partial\bar{Z}_I}{\partial t}+\mathbf{U}\cdot\nabla\bar{Z}_I+A_k\frac{\partial\bar{Z}_I}{\partial\xi_k}-B_{kl}\frac{\partial^2\bar{Z}_I}{\partial\xi_k\partial\xi_l}=\bar{W}_I.$$

Note that the negative sign of $\overline{\mathbf{N}}$ in (9) corresponds to the inverse diffusion term in the equation. Equation (5) is the Fokker–Planck equation for (1)–(3); derivations of the other MMC equations are given in Ref. 1.

In MMC, the closure is achieved by assuming that the reference PDF $P_{\xi}$ is known (typically Gaussian[1] or near-Gaussian[3]). The specification of the coefficients **U**, **A**, and **B** that complies with both the PDF $P_{\xi}$ and the average properties of turbulent scalar transport is given in Ref. 1. The minor dissipation time $\tau_S$, which does not affect $\bar{\mathbf{Z}}^*$ in conditional MMC but does affect $\mathbf{Z}^*$ in probabilistic MMC, was not specified in Ref. 1 and this specification requires special analysis considered in the present Brief Communication. In order to conduct this analysis, we first consider replacement of the reference variables $\boldsymbol{\xi}^*$ by a new reference variables $\tilde{\boldsymbol{\xi}}^*=\tilde{\boldsymbol{\xi}}(\boldsymbol{\xi}^*,\mathbf{x},t)$, the stochastic values $\tilde{\boldsymbol{\xi}}^*$ still represent a Markov process which satisfies all of the MMC equations given above but with the new drift $\tilde{A}_i$ and diffusion $\tilde{B}_{ij}$ coefficients,

$$\tilde{A}_i=\hat{D}(\underset{\mathbf{x}}{\mathbf{U}},\underset{\boldsymbol{\xi}}{\mathbf{A}},\underset{\boldsymbol{\xi}^2}{\mathbf{B}})\tilde{\xi}_i,\quad \tilde{B}_{ij}=B_{kl}\frac{\partial\tilde{\xi}_i}{\partial\xi_k}\frac{\partial\tilde{\xi}_j}{\partial\xi_l}. \tag{13}$$

Note that these equations simply represent the Ito differentiation rule for the stochastic process which is the time-reverse[7] of (1) and (2) and is equivalently specified by Eq. (7). One can note that, if the subset of the variables $\bar{\mathbf{Z}}$ which we denote as $\bar{\mathbf{Z}}_{\{i\}}$ (the major subset) represents a nonsingular replacement of the variables $\tilde{\xi}_i^*=\bar{Z}_i(\boldsymbol{\xi}^*,\mathbf{x},t)$ then $\tilde{A}_i=\bar{W}_i$ and $\tilde{B}_{ij}=\bar{N}_{ij}$. By itself, the mapping of $Z_I$ into $\bar{Z}_i=\tilde{\xi}_i$ does not represent a closure since the PDF of $\bar{Z}_i$ is unknown, but it is most convenient for the purposes of comparing the probabilistic and conditional versions of MMC pursued here.

The most simple form of MMC, which is considered in the present work, corresponds to a two-stream mixing problem with a single mixture fraction. The reference space is represented by a single variable $\xi$. In the rest of the Brief Communication, we operate with two scalars $Z$ and $Y$ selected from the complete scalar set $Z_1,Z_2,\ldots$. The scalar $Z$ is presumed to be the mixture fraction with the source term $W_Z=0$ and the minor dissipation operator $S_Z$ while $Y$ is any of the reactive scalars with the source term $W_Y$ and the minor dissipation operator $S_Y$. In conditional MMC, $\bar{Y}=\langle Y^*|\boldsymbol{\xi},\mathbf{x}\rangle$ can be expressed as a function of $\bar{Z}=\langle Z^*|\boldsymbol{\xi},\mathbf{x}\rangle$ while, in probabilistic MMC the stochastic values $Z^*$ and $Y^*$ would have some dispersion around $\bar{Y}=\bar{Y}(\bar{Z},\mathbf{x},t)$. We need to determine the value of the dispersion around $Q=\langle Y^*|Z,\mathbf{x}\rangle$ (the conditional variance) and compare it with its expected physical value. The asymptotic analysis presented below appears to be more simple with the following formal replacement of variables: $\xi$ is replaced by a new reference variable, $\tilde{\xi}=\bar{Z}(\xi,\mathbf{x},t)$. The replacement does not alter the closure but transforms the MMC equations into a new equivalent form that appears to be more convenient for analysis of the joint characteristics of the scalars $Y$ and $Z$. With the new notations, Eqs. (5) and (10) take the form

$$\hat{D}^{\circ}(\underset{\mathbf{x}}{\mathbf{U}},\underset{\bar{Z}}{\bar{A}^{\circ}},\underset{\bar{Z}^2}{\bar{N}},\underset{Z}{S_Z},\underset{Y}{\mathbf{W}+S_Y})P(Y,Z,\bar{Z},\mathbf{x};t)=0, \tag{14}$$

$$\hat{D}(\underset{\mathbf{x}}{\mathbf{U}},\underset{\bar{Z}^2}{\overline{\mathbf{N}}})\bar{Y}=\bar{W}. \tag{15}$$

Here and in the rest of the Brief Communication $\bar{Z}$ denotes the new reference variable $\tilde{\xi}$ and at the same time $\bar{Z}=\langle Z^*|\boldsymbol{\xi},\mathbf{x}\rangle$ due to the specific selection of $\tilde{\xi}$. Equation (15) demonstrates that the simplest form of the conditional MMC is equivalent to the first order CMC without the conditional variance effects, although MMC involves a consistent mixture fraction PDF method as a part of the model. Thus, the minor dissipation time $\tau_S$ cannot be determined on the basis of the average dissipation properties or the first order conditional characteristics—these conditions are already satisfied.[1] In order to determine $\tau_S$, we have to consider the conditional variance and investigate whether the properties not only of $Q=\langle Y^* | Z,\mathbf{x}\rangle$ but also $K=\langle (Y^*)^2 | Z,\mathbf{x}\rangle - Q^2$ modeled in the probabilistic MMC. It should be noted that, since both conditional and probabilistic MMC fully comply with the joint PDF equations, they must comply with all their consequences—the unclosed first and second order CMC equations.[4,8] This formal compliance, however, does not guarantee agreement with the actual CMC approach since the underlying MMC process is, obviously, not identical to physical turbulent mixing. Matching the conditional velocity/scalar correlation terms is achieved provided $\mathbf{u}_Z$ is a good model for the physical velocity conditioned on all scalars but the situation with the other generation and dissipation terms is much less clear and needs a special asymptotic analysis that establishes a link between $K$ and $\tau_S$ and is presented below.

At the first step, we derive equations for the moments of $P(Y,Z,\overline{Z},\mathbf{x};t)$ represented by $P_*=P(Z,\overline{Z},\mathbf{x};t)/P(\mathbf{x};t)$, $Q_* =\langle Y^* | Z,\overline{Z},\mathbf{x}\rangle$, and $K_*=\langle (Y^*)^2 | Z,\overline{Z},\mathbf{x}\rangle - Q_*^2$: Eq. (14) is multiplied by 1, $Y$, and $Y^2$ and the results are integrated over all $Y$

$$\hat{D}^\circ(\underset{\mathbf{x}}{\mathbf{U}},\underset{\overline{Z}}{\tilde{\mathbf{A}}^\circ},\underset{\overline{Z}^2}{\overline{N}},\underset{\overline{Z}}{S_Z})P_*\overline{\rho}=0, \tag{16}$$

$$\hat{D}(\underset{\mathbf{x}}{\mathbf{U}},\underset{\overline{Z}}{\mathbf{A}_*},\underset{\overline{Z}^2}{\overline{N}},\underset{\overline{Z}}{S_Z})Q_*=\frac{\overline{Y}-Q_*}{\tau_S}+W_*, \tag{17}$$

$$\hat{D}(\underset{\mathbf{x}}{\mathbf{U}},\underset{\overline{Z}}{\mathbf{A}_*},\underset{\overline{Z}^2}{\overline{N}},\underset{\overline{Z}}{S_Z})K_*=-\frac{2K_*}{\tau_S}+2\overline{N}\left(\frac{\partial Q_*}{\partial \overline{Z}}\right)^2+\Omega_*, \tag{18}$$

where

$$A_*=\tilde{A}^\circ-\frac{2}{P_*}\frac{\partial \overline{N}P_*}{\partial \overline{Z}},\quad \tilde{A}^\circ\equiv\frac{2}{P_{\overline{Z}}}\frac{\partial \tilde{N}P_{\overline{Z}}}{\partial \overline{Z}},$$

$\Omega_*\equiv\langle W^*\acute{Y}^* | Z,\overline{Z},\mathbf{x}\rangle$, $W_*=\langle W^* | Z,\overline{Z},\mathbf{x}\rangle$, and $\acute{Y}=Y-Q_*$.

We use the linear relaxation (4) for $S$ and introduce the small parameter $\varepsilon\equiv(\tau_S/\tau_d)^{1/2}$ and the characteristic variable $z=(Z-\overline{Z})/\varepsilon$, where $\tau_d$ is the characteristic physical dissipation time of the mixture fraction $Z$. Equations (16)–(18) take the form

$$\hat{D}^\circ\left(\underset{\mathbf{x}}{\mathbf{U}},\underset{z}{\frac{A_z^\circ}{\varepsilon^2}},\underset{z^2}{\frac{\overline{N}}{\varepsilon^2}},\underset{z}{\frac{-z}{\varepsilon\tau_d^{1/2}}}\right)P_*\overline{\rho}=0, \tag{19}$$

$$\hat{D}\left(\underset{\mathbf{x}}{\mathbf{U}},\underset{z}{\frac{A_z}{\varepsilon^2}},\underset{z^2}{\frac{\overline{N}}{\varepsilon^2}},\underset{z}{\frac{-z}{\varepsilon\tau_d^{1/2}}}\right)Q_*=\frac{\overline{Y}-Q_*}{\varepsilon^2\tau_d}+W_*, \tag{20}$$

$$\hat{D}\left(\underset{\mathbf{x}}{\mathbf{U}},\underset{z}{\frac{A_z}{\varepsilon^2}},\underset{z^2}{\frac{\overline{N}}{\varepsilon^2}},\underset{z}{\frac{-z}{\varepsilon\tau_d^{1/2}}}\right)K_*=-\frac{2K_*}{\varepsilon^2\tau_d}+2\overline{N}\left(\frac{\partial Q_*}{\partial \overline{Z}}\right)^2+\Omega_*, \tag{21}$$

where $A_z^\circ\equiv\tilde{A}^\circ\varepsilon-z/\tau_d$ and $A_z\equiv A_*\varepsilon-z/\tau_d$. The coefficients $\overline{N}$ and $\mathbf{U}$ depend on $\overline{Z}=Z-\varepsilon z$ and thus, to the leading order, are the functions of $Z$ but not $z$ while the leading orders of $A_z^\circ =-(z/\tau_d)+\cdots$ and $A_z=-(z/\tau_d)+\cdots$ depend on $z$. The leading order solution of (19) is given by the Gaussian distribution

$$P_0=G(z,0,\tau_d\overline{N}),$$

$$G(\zeta,\overline{\zeta},\sigma^2)\equiv\frac{1}{(2\pi\sigma^2)^{1/2}}\exp\left(-\frac{(\zeta-\overline{\zeta})^2}{2\sigma^2}\right). \tag{22}$$

Note that $P_0$ effectively approximates the conditional PDF $P_{Z|\overline{Z}}=P(Z|\overline{Z},\mathbf{x};t)$. The PDF $P_*$ can be represented by $P_* =P_0P_Z$ or by $P_*=P_0P_{\overline{Z}}$ (both representations are the same to the leading order). The value of $Q_*$ and its integrals are represented by the expansions

$$Q_*=Q_0(Z)+\varepsilon Q_1(Z)z+\varepsilon^2Q_2(Z)\frac{z^2}{2}+\cdots, \tag{23}$$

$$\overline{Y}(\overline{Z})=\int Q_*P_{Z|\overline{Z}}dZ=Q_0(\overline{Z})+\varepsilon^2\tau_d\overline{N}Q_{02}(\overline{Z}), \tag{24}$$

$$\overline{Y}(Z-\varepsilon z)=Q_0(Z)-\varepsilon Q_0'(Z)z+\varepsilon^2Q_0''(Z)\frac{z^2}{2}+\varepsilon^2\tau_d\overline{N}Q_{02}(Z), \tag{25}$$

where

$$Q_{02}(\overline{Z})\equiv\frac{Q_0''(\overline{Z})+2Q_1'(\overline{Z})+Q_2(\overline{Z})}{2}$$

and dependence on $\mathbf{x}$ and $t$ is implied in these equations. Substitution of the equations into (20) results in $Q_1=0$, $Q_2 =Q_0''/3$, and

$$\frac{\partial Q_0}{\partial t}+\mathbf{U}\cdot\nabla Q_0-\overline{N}Q_0''=W_*. \tag{26}$$

The "prime" and "double-prime" superscripts denote the first and second derivatives. The leading order term of the probabilistic MMC is consistent with the first order CMC equation (note that this consistency does not determine $\tau_S$). Equation (21) indicates that

$$K_*=\varepsilon^4\frac{\tau_d\overline{N}+z^2}{2}\tau_d\overline{N}Q_2^2,$$

$$K=\langle K_*\rangle_{\overline{Z}}+\langle Q_*^2\rangle_{\overline{Z}}-\langle Q_*\rangle_{\overline{Z}}^2=\frac{\tau_S^2}{c_\tau}\overline{N}^2\left(\frac{\partial^2 Q}{\partial Z^2}\right)^2, \tag{27}$$

where $\langle\cdot\rangle_{\overline{Z}}$ denotes averaging over $\overline{Z}$ by using $P_0$ and $c_\tau=6$. It should be noted that the value of the constant $c_\tau$ depends on the type the operator $S$ used in calculations.

The consistency of the model and the fast chemistry limit[9] is now considered. Assuming that, $Y$ is close to its chemical equilibrium, $Y=Y_e(Z)$, we put $P(Y,Z,\overline{Z},\mathbf{x};t) = \delta[Y_e(Z)-Y]P_Z P(\mathbf{x};t)$ in (14)

$$\frac{\overline{Z}-Z}{\tau_s}Y_e' = \frac{\overline{Y}-Y_e}{\tau_s} + W, \quad -\chi\frac{\partial^2 Y_e}{\partial Z^2} = W, \tag{28}$$

where $\chi \equiv (\overline{N}+z^2/\tau_d)/2$ is a model for the instantaneous scalar dissipation in (28) and the conditional mean $\overline{Y}$ is evaluated by formally putting $Q_0(Z)=\overline{Y}_e(Z)$ and $Q_1=Q_2=0$ in (23)–(25). Substitution of $\varepsilon z = Z-\overline{Z}$ and the expansion for $\overline{Y}$ given by (25) into the first equation in (28) yields the second equation. As expected, the average of $\chi$ over $\overline{Z}$ gives $\overline{N}$, although distribution of $z$ in $\chi$ is Gaussian but not log-normal. If $W=W(Z,Y)$ in (28) then this equation can be interpreted as a stationary flamelet equation[10] but, in its specification of the small-scale properties, the probabilistic model is not fully flamelet consistent: as soon as the reaction zone becomes thinner than the stochastic dispersion $\sigma_z$ in $Z$ space [which is specified by $\sigma_z^2=\tau_S\overline{N}$ in (22)], Eqs. (23)–(25) and (28) are no applicable.

The modeled value $K$ is to be compared with the corresponding value predicted by the CMC variance equation and DNS.[4,8,11,12] In the CMC variance equation, the major homogeneous generation term, which is induced by the fluctuations of scalar dissipation, can be approximated by $2\tau_N\overline{N}^2(Q'')^2$ while the dissipation term can be assessed as $2K/\tau_K$. While, with a single parameter $\tau_S$, simultaneous matching of the dissipation and the generation terms is not guaranteed, we, nevertheless, can match the required level of conditional fluctuations $K$. By comparing the steady-state value $K=\tau_K\tau_N\overline{N}^2(Q'')^2$ with (27) we conclude

$$\tau_S = (c_\tau \tau_K \tau_N)^{1/2}. \tag{29}$$

According to Ref. 11, $\tau_K$ is nearly four times smaller than the conventional dissipation time scale $\tau_d$ and $\tau_N$ is significantly less than the Kolmogorov time scale. Thus, indeed, as it has been assumed in the present analysis, $\tau_S$ should be selected small compared to $\tau_d$.

The first moment conditional properties of turbulence (and, consequently, the unconditional averages and dissipation) are well matched by the MMC models and this match is, generally, does not form a criterion for selecting the minor dissipation time $\tau_S$—one of the key MMC parameters. The main conclusion of the asymptotic analysis developed in the present work for the simplest version of the probabilistic MMC model with a single mixture fraction and a single reference variable is that $\tau_S$ should be determined by matching the level of conditional variance (i.e., fluctuations with respect to the averages of scalars, conditioned on a fixed value of the mixture fraction). The MMC model demonstrates a notable degree similarity with the expected values of the terms responsible for the generation of the conditional variances in turbulence[4,12] although the simulation of fluctuations of the scalar dissipation in the investigated simple version of MMC differs from the realistic characteristics of instantaneous dissipation. The obtained asymptotic equation for $\tau_S$ may need adjustments of its constant when different variations of the minor dissipation operator are used in the model.


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