Thermodynamics and time-directional invariance.

A. Y. Klimenko*

The University of Queensland, SoMME, QLD 4072, Australia

U.Maas

Karlsruher Institut fur Technologie, ITT, 76131, Germany

September 11, 2012

Abstract

Time directions are not invariant in conventional thermodynamics. We investigate implications of postulating time-direction invariance in thermodynamics and requiring that thermodynamic descriptions are not changed under time reversal accompanied by replacement of matter by antimatter (i.e. CPT-invariant thermodynamics). The matter and antimatter are defined as thermodynamic concepts without detailing their physical structure.

Our analysis stays within the limits of conceptual thermodynamics and leads to effective negative temperatures, to thermodynamic restrictions on time travel and to inherent antagonism of matter and antimatter. This antagonism is purely thermodynamic; it explains the difficulty in achieving thermodynamic equilibrium between matter and antimatter and does not postulate their mutual annihilation on contact. We believe that the conclusions of this work can be of interest not only for people researching or teaching thermodynamics but also for a wider scientific audience.

1 Introduction

1.1 Boltzmann's time hypothesis

In the 1890s, the kinetic theory of Ludwig Boltzmann, which represents an important link between thermodynamics and classical mechanics, attracted both interest and criticism. The criticism was to some extent motivated by doubts about the atomic (molecular) structure of matter, which were quite persistent at that time, but also involved a series of very interesting questions about consistency of the reversibility of classical mechanics with the irreversible nature of thermodynamics. Some of these questions (i.g. exact physical mechanism determining the direction of time) are not fully answered even today. In response to his critics, Boltzmann put forward a number of hypotheses of remarkable originality and depth [1, 2]. One of these hypotheses relates the perceived direction of time to the second law of thermodynamics and the temporal boundary conditions imposed on the universe. The consequence of this hypothesis is that, given different temporal boundary conditions, time may run in opposite directions in different parts of the universe. In other words, entropy can have an increasing trend in some sections of the universe and a decreasing in the other sections. This notion was introduced in the context of giant fluctuations occurring in the eternal universe, but the giant fluctuation hypothesis (which Boltzmann actually attributed to his assistant, Dr Schuets [1]) is the less interesting part of Boltzmann's analysis, which was designed to have his ideas understood by the scientific community of the 19th century. At that time, the prevalent understanding was that the Universe is eternal and any notion of initial conditions imposed at the beginning of the universe were not likely to be accepted as scientific.

---

*Email for communications: klimenko@mech.uq.edu.au

The essence of Boltzmann's time hypothesis is that the physical nature of time is direction-symmetric and so is the principal foundation of thermodynamics. The culprits of the observed irreversibility are the temporal boundary conditions imposed on matter in the universe or in the observed part of it (low entropy in the past and high entropy in the future, referring to the direction of time as we perceive it). The uneven boundary conditions result in the common trend of matter to have its entropy increase with time.

As the situation stands at present, we still do not know the exact physical mechanism that links the temporal boundary conditions (i.e. initial and final conditions) to the time-directional properties of matter. The temporal boundary conditions alone are not likely to be sufficient for explanation of the observed time asymmetry. The modern view, which perhaps is best articulated by Penrose [3], is that the physical laws should have a very small temporal directional bias, which is too small to be noticed in classical mechanics but the bias is dramatically amplified by ergodic mixing of dynamic trajectories (as studied in statistical mechanics). A tiny deviation from temporal symmetry, presumably related to decoherence of quantum states during their interaction with matter, causes an avalanche of the phase space increases and the flow of the thermodynamic time in one direction. The fundamental physical laws are close to but not fully time symmetric, although the symmetry is deemed to be exact if the opposite time directions are associated with matter and antimatter. At this point we refer to Feynman's theory of antiparticles [4], which treats antiparticles as particles moving back in time, and to CPT (charge/parity/time) symmetry in conjunction with the known minor CP violation [3]. The present work endeavours to construct a CPT-invariant thermodynamic theory, which in absence of antimatter is consistent with conventional thermodynamics. Note that all thermodynamic theories operating with scalar quantities are parity-invariant, while CT invariance is of most interest here.

1.2 Thermodynamic antimatter

While the exact mechanism of time and many other associated questions have yet to be answered by physical sciences, we are interested here in thermodynamical aspects of the Boltzmann time hypothesis. Following modern undestanding of this hypothesis combined with Feynman theory of antimatter, we assume that matter from our world, could be brought into thermodynamic contact with the so-called *thermodynamic antimatter* populating another section of the universe (one might call it the antiworld) where the time direction (i.e. the temporal direction of entropy increase) is opposite to ours. We use the term "thermodynamic antimatter" to follow the modern interpretation of the ideas of Ludwig Boltzmann and define a purely thermodynamic concept, which in principle may or may not be related to anti-particles and physical antimatter. Both matter and antimatter are presumed to be compliant with the first law of thermodynamics and have equivalent properties in reversible processes. However, matter and antimatter have opposite directions of irreversibility: isolated antimatter can decrease but cannot increase its entropy in our time.

Thermodynamic antimatter is a thermodynamic object that is postulated to comply with the following principles:

- **Reversible equivalence.** There is no distinction between matter and antimatter with respect to the first law of thermodynamics.

- **Inverted irreversibility.** Thermodynamically isolated antimatter can increase its entropy only backward in time (unlike any isolated matter, whose entropy increases forward in time).

- **Observational symmetry.** Antimatter and its interactions with matter are seen (i.e. observed, experimented with or measured) by antiobservers in exactly the same way as matter and its interactions with antimatter are seen by observers.

Conservation of energy by the reversible equivalence principle also implies preservation of mass. Hypothetical observers made of antimatter, are called *antiobservers,* while the term *observers* refers

only to us — observers made of matter. Properties of matter measured by us (the observers) and properties of antimatter measured by antiobservers are referred to as *intrinsic.* The properties of matter and antimatter measured by observers are referred to as *apparent,* while the properties of matter and antimatter measured by antiobservers are referred to as *antiapparent.*

Despite the relative simplicity of our assumption and analysis, our conclusions involve an inherent thermodynamic hostility of matter and antimatter and seem to deviate from existing views. The suggested approach is different from the conventional assignment of the same irreversible thermodynamic properties to both matter and antimatter implying violations of the CPT symmetry [3]. The competing physical intuitions of CPT invariance and of conventional thermodynamics has been recently discussed by Downes et al [5]. Our approach is also different from thermodynamic analysis of physical antimatter by Dunning-Davies [6], which is based on Santilli isodualities: our reversible equivalence principle does not need anti-photons and antigravity that are associated with Santilli isodualities and remain seen by most physicists as unlikely. Our approach based on thermodynamic antimatter seems to suggest some alternative astrophysical interpretations. For example, in his fundamental work [7], Hawking compared thermodynamics of black and white holes and concluded that black and white holes of the same mass should have the same temperatures. Our inference is that, assuming that the thermodynamic antimatter is real and thus should form white holes, the effective temperature of white holes should be much higher.

While mentioning similarities and differences with a number of existing physical theories, we should state that our approach is based on thermodynamics — it operates with thermodynamic concepts and objects, which are defined by postulating their thermodynamic properties.

2   Thermodynamics of interactions of matter and antimatter

Thermodynamics is based on determining the direction of processes where states (i.e. macrostates) can be realised by the largest possible number of microstates (given the constraints imposed on the system) and thus are overwhelmingly more likely than states encompassing fewer microstates. The logic of thermodynamics considers what is likely and neglects what is unlikely. The most likely state is called equilibrium. In conventional thermodynamics, unlikely states may be set as initial states while the system tends to move towards its equilibrium as time passes. This is reflected by the well-known Boltzmann–Planck entropy equation

$$S_i = k_B \ln(W_i) \tag{1}$$

linking the entropy S_i in the state i to the number of microstates W_i in this state. The number of microstates W_i is further referred to as the statistical weight of the thermodynamic state i. The constant k_B is the Boltzmann constant that rescales very large changes in W_i to more manageable quantities of conventional thermodynamics.

The most difficult part in considering time-symmetric behaviour is the necessity to suspend the causality principle, which is deeply engraved in our way of thinking. This necessity was clearly stipulated by Price [8] who has thoroughly discussed philosophical aspects of the direction of time. Indeed, conventional thinking, which is based on causality and allows the influence of the past on the future but not vice versa, is inherently time-asymmetric and can not be used for considerations that are neutral with respect to the direction of time. In conventional thermodynamics, the likeness of the future states is evaluated for a selected time moment in the future, given fixed conditions specified in the present. This has to be replaced by direction-neutral evaluation of the likeness of different time trajectories, consistent with the boundary conditions, which are fixed by external means, and other physical laws and constraints. Specifically, the conditions for matter are fixed in the past while conditions for antimatter are fixed in the future (from our perspective and in the past from the antiobserver perspective). According to the declared observational symmetry principle, our analysis should remain symmetric with respect to time: we (i.e. the observers) see antimatter in exactly the same way as the antiobservers see our matter.

2.1 Apparent temperatures

The temporal boundary conditions for the example shown in Figure 1 are: U_m, S_m are specified for matter at $t = -t_0$ and $\bar{t} = t_0$ and $\bar{U}_m, \bar{S}_m$ are specified for antimatter at $\bar{t} = -t_0$ and $t = t_0$. The overbar symbol indicates that the value is antiapparent, i.e. evaluated from the perspective of an antiobserver, whose time $\bar{t} = -t$ goes in the opposite direction as compared to our time t. A limited thermodynamic contact of matter and antimatter, allowing for transition of a small quantity of thermal energy δQ, occurs at $t = 0$ (and $\bar{t} = 0$). According to observer the thermal energy δQ is transferred from antimatter to matter as shown by the black solid arrow. According to the antiobserver, who has the opposite direction of time the same thermal energy δQ is transferred from matter to antimatter as shown by the red dashed arrow. Heat δQ is assumed to be positive when transferred in the direction shown in Figure 1: from antimatter to matter according to the observer and from matter to antimatter according to the antiobserver. The total energy

$$U_{tot} = U_m + \left(\bar{U}_a + \delta Q\right) = (U_m + \delta Q) + \bar{U}_a \tag{2}$$

(evaluated at any constant time $t = -\bar{t}$) is preserved in this example, as it should since the formulation of the first law of thermodynamics does not depend on the differences between matter and antimatter due to the postulated reversible equivalence. The entropy change of matter as observed by us and the entropy change of antimatter as observed by the antiobserver (these are the entropies linked to W) can be easily evaluated and these changes of intrinsic entropy are shown in Figure 1 for the states m' and a'.

We now evaluate the overall statistical weight W_{tot} that corresponds to different trajectories that are allowed by the first law of thermodynamics. The overall state is related to the four sub-states: m, a, m' and a'. The overall statistical weight W_{tot} is linked to the product of the statistical weights of the sub-states $W_m W_{m'} W_a W_{a'}$ and, according to equation (1), becomes

$$W_{tot}(\delta Q) \sim W_m W_{m'} W_a W_{a'} = \exp\left(\frac{S_m + S_{m'} + \bar{S}_a + \bar{S}_{a'}}{k_B}\right) \tag{3}$$

The value W_{tot} depends on δQ. Note that S_m and $\bar{S}_a$ are fixed by the boundary conditions and only $S_{m'}$ and $\bar{S}_{a'}$ depend on δQ. In this example, we should place the time moments $t = -t_0$ and $t = +t_0$ as far apart as needed to ensure establishment of equilibriums within matter and antimatter before and after the interaction, which is used in (3) in form of stochastic independence of microstates that correspond to the macrostates m and m', a' and a. If this was not the case, then not all microstates of state m' may be accessible due to stochastic correlations with state m (in simple terms the system may not have enough time to establish its equilibrium state). Assessing the statistical weight of a continuous trajectory is, generally, a difficult task due to possible correlations between microstates. Indeed, in conventional thermodynamics, a system which is initially placed into a non-equilibrium state, is unlikely to be transferred in the next moment into any microscate that belongs to the equilibrium state of the system. However, as sufficient time passes by, all microstates becomes achievable and the system can be found in its equilibrium state with overwhelming probability. Here, we place the time moments $t = -t_0$ and $t = +t_0$ sufficiently far apart so that the system microstates at these moments do not correlate. Two of the states, m and a, are fixed by the boundary conditions.

Equation (3) can be simplified through normalising W_{tot} by the value of W_{tot} at $\delta Q = 0$

$$\frac{W_{tot}(\delta Q)}{W_{tot}(0)} = \exp\left(\frac{\delta Q}{k_B}\left(\frac{1}{T_m} + \frac{1}{\bar{T}_a}\right)\right) \tag{4}$$

where conventional definitions of the temperature

$$\frac{1}{T_m} = \frac{\partial S_m}{\partial U_m}, \quad \frac{1}{\bar{T}_a} = \frac{\partial \bar{S}_a}{\partial \bar{U}_a} \tag{5}$$

4

are used. (The quantity δQ is assumed to be too small to affect the temperatures of matter and antimatter, which remain T_m and $\bar{T}_a$ correspondingly.) It should be noted that identical intrinsic temperatures of matter and antimatter $T_m = \bar{T}_a$ do not ensure equilibrium between them, since transferring energy from antimatter to matter (from our perspective, and from matter to antimatter from the perspective of the antiobserver) increases the overall statistical weight and is strongly favoured by thermodynamics. If matter and antimatter are to be placed in conditions of thermodynamic equilibrium, both directions of heat transfer $\delta Q > 0$ and $\delta Q < 0$ must be equally likely and have the same statistical weight W_{tot}. This occurs only when $T_m = -\bar{T}_a$ indicating that the apparent temperature of antimatter is $T_a = -\bar{T}_a$ so that the conventional equilibrium condition in our frame of reference is given by $T_m = T_a$. In the same way, the antiapparent (i.e. perceived by the antiobserver) temperature of matter is $\bar{T}_m = -T_m$. It is easy to see that thermodynamic quantities $\bar{S}_a$, $\bar{T}_a$, and $\bar{U}_a$ that characterise the intrinsic properties of antimatter are apparent as

$$T_a = -\bar{T}_a, \;\; S_a = -\bar{S}_a, \;\; U_a = \bar{U}_a \tag{6}$$

from our perspective. The sign of U_a is selected to be consistent with the first law of thermodynamics (2), while the sign of S_a is chosen to be consistent with the definition of temperature $T_a^{-1} = \partial S_a / \partial U_a$ and with equations (5). The change of sign does not affect our interpretation of reversible transformations of antimatter since S_a is constant whenever $\bar{S}_a$ is constant, which is consistent with our assumption that matter and antimatter behave in the same way in reversible processes.

It appears that negative temperatures created in our world can, at least in principle, be placed into thermal equilibrium with thermodynamic antimatter, which is controlled by reverse causality. If, according to the Boltzmann hypothesis, the direction of time and causality are determined by the action of the second law of thermodynamics, then one might expect the existence of a degree of reverse causality for matter in states with negative temperatures.

2.2   Apparent heat capacities

Equation (6) indicates that thermodynamic considerations of antimatter involve not only negative temperatures but also negative heat capacities. Indeed, the equation linking the heat capacity C to S, U and T (for example, see [9])

$$C_a = \frac{-1}{T_a^2} \left(\frac{\partial^2 S_a}{\partial U_a^2} \right)^{-1} = \frac{1}{\bar{T}_a^2} \left(\frac{\partial^2 \bar{S}_a}{\partial \bar{U}_a^2} \right)^{-1} = -\bar{C}_a \tag{7}$$

indicates that changing the sign of the entropy S changes the sign of the heat capacity C irrespective of the sign of the temperature T. Negativeness of antimatter heat capacities is illustrated in Figure 2. The heat capacity of antimatter is positive according to the antiobserver and negative according to the observer. As seen by the antiobserver, two thermodynamic antimatter objects with initial intrinsic temperatures $\bar{T}_1$ and $\bar{T}_2$ at $\bar{t} = -t_0$ are brought into contact. The second object is presumed to be somewhat intrinsically hotter than the first objects $\bar{T}_2 > \bar{T}_1$ so that heat $\Delta Q > 0$ is transferred, according to an antiobserver, from the second object to the first object to reach the equilibrium temperatures $\bar{T}_0$ at $\bar{t} = +t_0$. Note that $\bar{T}_2 > \bar{T}_0 > \bar{T}_1 > 0$ and $\bar{C}_1, \bar{C}_2 > 0$, while the observational symmetry principle requires that, according to any antiobserver, the thermodynamic properties of antimatter are the same as thermodynamic properties of matter observed by us.

The same process looks different from our (the observer's) perspective. The initial equilibrium state with temperature T_0 at $t = -t_0$ is replaced by non-equilibrium states $T_1 > T_2$ at $t = +t_0$ due to transferring heat from the first body to the second body. One can easily see that the condition $T_1 > T_2$ does not allow for positive heat capacities C_1 and C_2. This explains why positive heat capacities $\bar{C}_1, \bar{C}_2 > 0$ seem negative $C_1, C_2 < 0$ to us. The initial equilibrium state detected by the observer is unstable — when the heat capacities are negative, a small amount of heat δQ transferred from the first object to the second object increases the apparent temperature of the former and

5

decreases the apparent temperature of the latter. This stimulates further apparent transfer of heat from hotter to colder objects, i.e. in the same direction. Instability of equilibrium states is a general property of negative heat capacities as considered in the following section. Note that both the observer and the antiobserver determine that the process is consistent with the second law as seen from their respective perspectives — the heat is transferred from hotter to colder objects. The possibility of thermodynamic description backwards in time comes as the cost of introducing unusual "negative" thermodynamics. We stress that our prime goal is not in a rather formal task of interpreting heat transport backward in time but in analysis of a more complex and interesting problem — interactions of matter and antimatter.

2.3 Mass exchange between matter and antimatter

An antimatter system with a variable number of particles is characterised by the equation

$$d\bar{U}_a = \bar{T}_a d\bar{S}_a + \bar{\mu}_a d\bar{N}_a \tag{8}$$

which remains conventional as long as it is presented from the perspective of the antiobserver. From our perspective, this equation changes according to (6). The reversible equivalence requires preservation of mass, which demands that the apparent and intrinsic numbers of particles composing antimatter are the same $N_a = \bar{N}_a$. Hence $\mu_a = \bar{\mu}_a$.

Consider a reaction converting matter to into antimatter (one-to-one to preserve the total mass) under conditions when the energy of matter and antimatter remain the same $dU_m = 0$ and $dU_a = 0$. If dN particles have been converted from antimatter to matter, the change in total apparent entropy is given by

$$dS_{tot} = dS_m + dS_a = -\frac{\mu_m}{T_m}dN_m - \frac{\mu_a}{T_a}dN_a = -\left(\frac{\mu_m}{T_m} + \frac{\bar{\mu}_a}{\bar{T}_a}\right)dN \tag{9}$$

where we take into account that $dN_m = +dN$ and $d\bar{N}_a = dN_a = -dN$ and that $\bar{T}_a = -T_a$. Assuming that $\bar{T}_a = T_m$, the observational symmetry principle requires that $\bar{\mu}_a = \mu_m$ resulting in

$$dS_{tot} = -2\frac{\mu_m}{T_m}dN \tag{10}$$

Hence thermodynamics strongly favours transfer from antimatter to matter if $\mu_m < 0$ and vice versa if $\mu_m > 0$ (according to the observer). In this case equivalence of the observed chemical potentials $\mu_a = \mu_m$ does not ensure equilibrium of the matter/antimatter reaction, since the corresponding apparent temperatures $T_a \neq T_m$ are not at equilibrium. If, however, matter and antimatter are somehow brought into thermal equilibrium and have the same apparent temperatures $T_m = T_a$, the chemical potentials μ_m and $\mu_a = \bar{\mu}_a$ would not coincide since matter and antimatter must have very different properties at very different intrinsic temperatures T_m and $\bar{T}_a = -T_m$. The properties of matter and antimatter are determined by their intrinsic parameters, while equilibrium between matter and antimatter is controlled by apparent (or antiapparent) values of the parameters.

3 "Negative" thermodynamics

In this section we briefly review general features of systems with negative temperatures and negative heat capacities, which are not necessarily related to the apparent properties of antimatter and have been repeatedly discussed in the literature for other applications [10, 11, 12, 13, 9]. This section is presented from the observers perspective.

3.1 Negative temperatures

In this section we discuss general properties of negative temperatures, which may or may not be related to reversal of the direction of time and matter/antimatter interactions. In our world, negative temperatures can appear in systems with a limited number of microstates at high energies [10]. A very simple example of a system with negative temperatures can be found in Ref. [9] The effects encountered in lasers and, possibly, in biological organisms bear resemblance to negative temperatures. Practically, negative temperatures are difficult to create and even more difficult to maintain since systems with negative temperatures tend to be unstable [11].

Unlike the mathematical convention of ordering positive and negative numbers, negative temperatures are not lower but higher than the positive temperatures [10, 9]. The lowest possible temperature is $T = +0$. Positive temperatures can increase up to $T = +\infty$, which is the same as $T = -\infty$ and can be interpreted as the effective temperature of work. Negative temperatures can increase further up to $T = -0$, which is the highest possible temperature. Is is difficult to remove heat from low temperatures and add heat to high temperatures. One can see that temperature ordering is consistent with conventional mathematical ordering of the following quantity

$$\beta = -\frac{1}{T} \tag{11}$$

which we can call quality of energy or quality of heat. The quality of work corresponds to $\beta = 0$. Conventional heat with positive temperatures is of lower quality $\beta < 0$, while heat with negative temperatures is of higher quality $\beta > 0$ than work. Note that infinite temperatures do not necessarily need to posses infinite energies and can be linked to a loss of the dependence of the number of stochastic degrees of freedom present in the system on energy. Energy without a random component, i.e. work or coherent light, are assigned the values of $\beta = 0$ and $T = \infty$.

The second law of thermodynamics indicates that the overall quality of heat cannot increase in an isolated system. While the quality of heat can decrease in an isolated system, this process is irreversible as subsequent increases of the heat quality are not permitted. For example, as shown in Figure 3(a) transition of heat δQ from β_2 to β_1 is possible but it is irreversible since transition δQ from β_1 to β_2 is prohibited by the second law. Any reversible change of heat quality needs to involve at least three different levels to preserve the overall quality of heat. The reversibility condition for the process shown in Figure 3(b) is thus

$$\beta_0 \left(\delta Q_H + \delta Q_L \right) = \beta_H \delta Q_H + \beta_L \delta Q_L \tag{12}$$

that is the increase of the heat quality from β_0 to β_H must be compensated by the decrease of the heat quality from β_0 to β_L. Equation (12) preserves the overall entropy and is valid for the whole range of $-\infty < \beta < +\infty$.

Any engine is designed to produce work, hence one of its energy qualities that corresponds to the produced work is $\beta = 0$. A hypothetical reversible engine producing work in most efficient way allowed by the second law is conventionally referred to as Carnot engine. As a reversible engine, Carnot engine must comply with condition (12). There are three modes that a Carnot engine can operate in, as shown in Figure 4. The first is the conventional Carnot engine working with positive temperatures, where the quality of heat δQ_{H1} is increased from $\beta_{H1} < 0$ to work $\beta = 0$ with compensating decrease of the quality of heat δQ_{L1} from β_{H1} to β_{L1}. The second engine works with negative temperatures: the quality of heat δQ_{L2} is decreased from $\beta_{L2} > 0$ to work $\beta = 0$ which gives the opportunity to increase quality of some of the heat δQ_{L2} from β_{L2} to β_{H2}. The third engine works with one positive and one negative temperature heat reservoirs and manages to produce work from both of these sources. We stress that this does not contradict the second law since the heat quality increases from β_{L3} to $\beta = 0$ and decreases from β_{H3} to $\beta = 0$ while the overall quality of energy remains the same.

3.2 Negative heat capacities

Although not common in conventional thermodynamics, negative heat capacities have been repeatedly discussed in the literature [12, 13, 9]. From the point of view of conventional thermodynamics, negative heat capacities $C < 0$ are even more unusual than negative temperatures $T < 0$. A thermodynamic system with negative C can not be divided into equilibrated subsystems and its thermodynamics can not be evaluated with the use of partition functions [9]. Indeed, if a system consists of two equilibrated systems 1 and 2 with $C_1 < 0$, $C_2 < 0$ and $T_1 = T_2$, this equilibrium is unstable since transferring $\Delta Q > 0$ from system 2 to system 1 increases T_2 and decreases T_1, which further stimulates heat transfer in the same direction. Stars and black holes may serve as realistic examples of systems with negative heat capacities [12, 9].

Interactions of systems with negative and positive heat capacities are capable of unusual behaviour that seemingly violates the second law of thermodynamics, but in fact the average quality of energy in these interactions stays the same or decreases and the entropy does not decrease. Consider the example shown in Figure 5. Interactions of two systems with $-C_2 = C_1 > 0$ may result in an apparent increase of their temperatures through reversible hear transfer ΔQ occurring between the same temperatures. Note that, according to the definition of heat capacity

$$\frac{\partial S}{\partial T} = \frac{C}{T} \tag{13}$$

the entropy S_2 decreases in this process by the same amount as the entropy S_1 increases (assuming $T > 0$) and the overall quality of thermal energy remains the same. Since the process is reversible when the absolute values of the heat capacities are the same, apparent decrease in the temperatures due to negative ΔQ is also possible. If, however, $|C_2| < |C_1|$, the process of temperature increase becomes irreversible since T_2 grows faster than T_1 as shown in Figure 5. In this case the overall entropy $S_1 + S_2$ increases and some of the quality of heat is lost.

4 Examples

This section offers a number of gedanken experiments that are designed to illustrate the properties of thermodynamic antimatter. While the overall setup of these experiments is, of course, unrealistic, the points illustrated by them may well be quite relevant to the real world.

4.1 On shaking hands with anti-people

First we recall the well-known warning of Richard Feynman (based on CP invariance) not to shake alien left hand if this hand is offered for the handshake — the alien might be made from antimatter. We act cautiously and decide to send a robot to perform this handshaking mission. Everything begins from a radio message arriving from the outer space thanking us for sending our robotic representative to shake hands with their robotic representatives at a given very remote location — this encounter was most educational. We look at that location through a telescope and see what seem to be remaining of a big explosion but still decide to send a robot there. The robot is instructed to offer a handshake at certain location in space and time. As our robot stretches his robotic arm, another robot (an antirobot, as the reader has already guessed) materialises from the surrounding dust cloud and, for an instant, shakes the stretched hand (Figure 6). In accordance with the thermodynamics of antimatter, our robot receives a very large amount of energy from the antirobot, which blows him into small pieces. This effect is purely thermodynamic (the same as shown in Figure 1) — it does not need annihilation of physical matter and antimatter occurring on their contact. The alien antirobot then retreats backwards into his alien antiworld, while we do not have any realistic chance of reassembling our robot back. As polite people, we do not forget to send a radio message thanking our counterparts for the educational encounter.

Could this meeting be more productive without shaking hands? It certainly could, but we must keep in mind that many other accidents are thermodynamically possible during the meeting. For example, as the robots approach each other, they might be hit by a coherent light beam. While our robot reports being hit by the beam radiated by the alien robot, his counterpart makes exactly the same compliant. These complaints miss the main point that the beam converting energy from negative temperatures ($\beta > 0$) into coherent radiation ($\beta = 0$) and from coherent radiation into positive temperatures ($\beta < 0$) is strongly favoured by thermodynamics. Whether the beam actually occurs or not is determined by the kinetics of the process and, as all kinetic issues, is outside the scope of our analysis.

4.2   Travelling to the antiworld

A prudent traveller to distant places must take enough fuel to provide a continuous supply of energy but this is going to be the least difficult part for our planned journey. Any object found in the antiworld would represent a source of energy of very high quality. Here we do not need to interpret antimatter as composed of antiprotons and positrons, which can readily react with protons and electrons to release plenty of energy (although this indeed might be the case). The energy is provided thermodynamically by antimatter releasing large amounts of energy as soon as we come into a thermodynamic contact with it. This indicates that the antiworld is a rather dangerous place for us. Specifically we should keep away from any objects we might see there, anti-stars and anti-planets (one may note that anti-planets are hotter for us than anti-stars). Antiobservers populating the antiworld would see us as excessively hot and dangerous.

The question if we would be able see anything in the antiworld is reasonable question to ask. The temperatures of anti-stars $T \approx -6000K$ is well below that of background radiation (combined with some dispersed antimatter), which is extremely hot at $T \approx -3K$ (if we use the familiar conditions from our world as a model for the antiworld). Thus transfer of radiative energy is directed from background to the stars. The hottest object we may find in the antiworld are black holes, which we see as white holes due to reversal of time. The temperature T of large white holes approaches $-0K$ and almost nothing can enter its horizon. An antimatter object at $T \approx -3K$, although very hot, may be less dangerous for us than another "cooler" object at, say $T \approx -6000K$, since the first object may have very little energy left to pass onto us during a contact (while the object keeps increasing its temperature from $T \approx -3K$ to $T \approx -0K$).

The most interesting part in our journey is, of course, not interpreting conventional thermodynamics backward in time but the interactions of antimatter (antiworld) and matter (us). The prospects of this interaction are not particularly soothing. The largest problem we will have to face is not lack of energy but an excess of it. The antiworld around us is so hot that we will not be able to dump excess heat anywhere. The only opportunity we have is delaying our imminent destruction due to overheating by having a perfectly reflecting coating on our space ship and taking plenty of ice for cold drinks with us. If we stay away from antimatter and its radiation, we might be lucky to stretch our journey little bit longer in our time.

In any case, we conclude that the antiworld is a very hostile environment for matter and traveling there is not recommended. The same travel warning applies to any attempts to build a time machine that allows us to move back in time but is not capable of complete thermodynamic separation of the time machine from the rest of the world. Note that matter moving back in time acquires thermodynamic properties of antimatter. We should not blame the antiworld for its hostile attitude towards us as our world is no less hostile towards visitors from antiworld and an anti-traveller would encounter the same problems in our world as we encounter in his. In the conditions currently prevailing in the universe, the opportunities for existence of macroscopic quantities of thermodynamic antimatter are very limited.

4.3 "Thermodynamic Bang"

The process of having thermodynamical matter, antimatter and radiation in equilibrium at infinite temperature with subsequent cooling is called here the thermodynamic bang. This, of course is not a model for the real Big Bang but an illustration of direction of the processes that may thermodynamically occur under specified conditions (as indicated by reactions I, II and III in Figure 7).The division of the cooling process into stages is purely schematic and given as illustration for the direction of the processes for each of the reaction, while overall evolution of the system is determined by reaction kinetics with all three reactions occurring at the same time.

The bang. The point with $T_m = \bar{T}_a = \infty = T_a$ (i.e. $\beta_m = \bar{\beta}_a = 0 = \beta_a$ where $\beta = -1/T$) is assumed to be located at $t = \bar{t} = 0$ as shown in Figure 7. When the temperatures are close to infinity, the system is dominated by radiation but may include matter and antimatter in the state of thermodynamic equilibrium. The chemical potentials of radiation, matter and antimatter are conventionally set to zeros.

Stage I. As the system cools adiabatically due to expansion in the direction of positive t, matter $\mathfrak{m}$, antimatter $\mathfrak{a}$ and radiation $\mathfrak{r}$ can react according to

$$\text{(I) } \mathfrak{r} \rightleftarrows \mathfrak{m} + \mathfrak{a} \tag{14}$$

producing more matter and antimatter from radiation. Assuming that adiabatic expansion is reversible and that there are no thermodynamic interactions between matter and antimatter, the intrinsic temperatures of matter and antimatter must be the same $T_m = \bar{T}_a$ (that is $\beta_m = \bar{\beta}_a$) and decreasing. This corresponds to negative apparent temperatures of antimatter $T_a = -\bar{T}_a < 0$ (and $1/\beta_a = -1/\bar{\beta}_a > 0$). Under these conditions reaction (I) can occur reversibly as illustrated in Figure 3(b), although produced matter and antimatter are not in thermal equilibrium with each other since they have different apparent temperatures T_m and $T_a = -T_m$. The temperature of the radiation, which can not be equilibrated by matter or antimatter, remains effectively infinite (if the notion of temperature can be applied to radiation at this stage). The chemical potentials of matter and antimatter are the same $\mu_m = \mu_a = \bar{\mu}_a$ due to the observational symmetry but do not have to take zero values since reaction (I) is not at its thermodynamic equilibrium and the condition $\mu_r = \mu_m + \mu_a$ does not apply. Expansion and loss of pressure is likely to make $\mu_m = \mu_a = \bar{\mu}_a$ negative.

Stage II. The thermodynamic interactions of matter and antimatter result in heat being transferred from antimatter to matter according to the equation

$$\text{(II) } (U_a + \Delta Q) + U_m \rightleftarrows U_a + (U_m + \Delta Q) \tag{15}$$
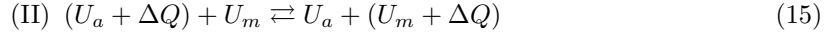
reheating matter and, possibly, stimulating inflational expansion. The apparent temperatures increase similar to the mechanism shown in Figure 5. The heat transfer specified by (II) is irreversible since $T_a > T_m$.

Stage III. If the following reaction is allowed

$$\text{(III) } \mathfrak{a} \rightleftarrows \mathfrak{m} \tag{16}$$

antimatter is then converted into matter (and, possibly, also annihilate with matter into radiation according to reaction (I)). Note that reaction should preserve the intrinsic temperature $T_m^\circ = \bar{T}_a$ due to conservation of energy and similarity of matter and antimatter. Hence reaction (II) acts to cool matter and the rest of the system since the temperature T_m° of the converted matter is below the temperature T_m of the existing matter. Assuming negative chemical potential of matter, reaction (III) increases the entropy due to equation (10). Both reactions (II) and (III) are thus irreversible and predominantly proceed in the forward direction. Finally, as antimatter disappears, interactions of matter and radiation establish equilibrium between them. From this moment, conventional forward-time causality is established and antimatter may exist only as a thermodynamic fluctuation.

The same process may occur backward in time establishing dominance of antimatter and inverse causality. Thus the antiworld is located "before" the bang in our time and there is no way for us to

access it. According to the observational symmetry principle, anti-people populating the antiworld would say exactly the same things about us. Disappearance of antimatter at $t > 0$ converts the initial time-elliptic equilibrium state into highly non-equilibrium state with directional time. The stated negativeness of the chemical potentials $\mu_m = \mu_a = \bar{\mu}_a$ is essential for the suggested scheme. If we assume $\mu_m = \mu_a = \bar{\mu}_a > 0$ antimatter is going to be dominant at $t > 0$ and matter is going to be dominant at $t < 0$, then our "thermodynamic bang" becomes a "thermodynamic crunch".

In our analysis, it is important that the time coordinate is not looped, although closed time-like loops are formally possible in general relativity. Time loops are likely to stimulate antimatter production. Indeed, if time forms a loop, then the production of matter in one direction is not separated from the production of antimatter in the opposite direction. A prudent space traveller should thus beware of any temporal Merry-go-round if he does not want to experience an "antimatter shower".

The assumptions made in this paper necessitate that, in absence of matter and antimatter, radiation can not change its entropy. In our world, interactions of radiation and matter result in increase of the overall entropy of matter and radiation forward in time. In the same way, interactions of radiation and antimatter in absence of matter result in increase of the overall entropy backward in time. If both matter and antimatter are present in large quantities, radiation should have infinite temperature (i.e. stay coherent). We also should note that no black/white holes can be formed in this mixture. Indeed, matter forms black holes while antimatter forms white holes due to corresponding forward and backward irreversibilities of these processes. Can a hole be black and white at the same time? The exterior of a black hole is no different from that of a white hole. Inside the horizon, both holes have the same metric but with a different direction of the time-like coordinate: the singularity is in the future of black holes and in the past of white holes. Thus, a hole may switch from white to black but can not be white and black at the same time (i.e. under the same horizon). Hence, the mixture of matter and antimatter can not form black/white holes even if matter and antimatter gravitationally attract each other. This might be the possible driving factor that ensures perfect uniformity of the bang — it is just cannot exist in any other way. The time is non-directional near the bang point $t \sim 0$, which means by definition that there are no white or black holes since these holes are time-directional.

5   Discussion

While this work was started as an entertaining thermodynamic exercise, it seems now that it has implications stretching beyond our original intentions. The presented consideration connects the perceived direction of time to abundance of matter in the universe, to thermodynamic antagonism of matter and antimatter and to the fact that, as far as we know, time does not form a closed loop. Although it is always possible to justify the arrow of time by introducing causality, this corresponds to postulating direction of time without explaining its physical mechanism [8, 14] (and there must be one).

The Boltzmann time hypothesis is enlightening in many respect and the temporal boundary conditions imposed on the universe are important but, when taken alone, insufficient to explain why causality is routinely observed at almost every scale. Matter should have at least some time-directional bias, which must be very small as we do not detect it in both classical mechanics and conventional quantum mechanics. The bias, nevertheless, has a profound effect on the universe. Penrose [3] provides with a number of convincing arguments supporting this view but, ultimately, the exact physical mechanism of running time is yet to be found. Considering that, to the best of our knowledge, the world is CPT-symmetric and very close to CP-symmetric, matter should be in a minor violation of the time symmetry. The apparent problem comes from antimatter and from the Feynman theory [4], which treats antiparticles as moving back in time. If matter has a slight forward time bias, the CPT symmetry requires antimatter to have the same bias backward in time. How this can be consistent with the conventional second law[5] ? Our CPT-invariant thermody-

namics seems to resolve this issue. We stress, however, that this paper introduces a consistent thermodynamic theory constructed on postulated properties of thermodynamic antimatter, while the degree of its applicability to real antimatter is yet to be determined. This investigation is by far not trivial since the exact mechanism enacting the direction of time is not known and must be very fine while intrinsic thermodynamic properties of small quantities of antimatter can be overpowered by interactions with matter–populated outside world, even if these interactions remain very weak. At present, the apparent absence of large quantities of antimatter in the universe gives the best support to the theory.

In absence of antimatter, the CPT-invariant thermodynamics should coincide with conventional thermodynamics and, generally, this is the case. Has any conventional thermodynamic property been lost in the introduced thermodynamics? Yes, there is one — the photon gas (which is the same as anti-photon gas) has lost its ability to independently comply with the second law. Consider gedanken experiment suggested by Hawking [7]: either radiation or matter and radiation are placed in a box (which may reach a galactic scale and contain black holes) with perfectly reflective insulating boundaries (mirrors). There is an expectation that a mixture of matter and radiation will eventually come into equilibrium. Our thermodynamics agrees with this. However, if radiation is present in the box without matter, its evolution according to our theory becomes time reversible, its entropy does not increase and the gas can not be equilibrated. Is this a problem? We would argue that it is not, since either the box mirrors are made of matter and are not perfectly reflective or this experiment can hardly be conducted in any practical form or shape. Thermodynamic equilibration of the pure photon gas in a universe, which is free of any matter and antimatter, is not an experimental fact but a hypothesis primarily based on validity of the conventional second law of thermodynamics.

6   Conclusions

The present work follows modern understanding of the Boltzmann time hypothesis and Feynman concept of antimatter as being similar to matter but moving backward in time. The developed thermodynamics of matter/antimatter interactions is both CPT-invariant and irreversible at the same time. The forward-time irreversibility is seen as the property of matter populating the world, which is accompanied (at least in principle) by backward-time irreversibility of thermodynamic antimatter. Matter and antimatter, however, display the same properties in reversible processes. Our direction-neutral consideration of thermodynamics is necessarily accompanied by suspending the causality principle. The analysis of thermodynamic interactions of matter and antimatter leads to negative temperatures: the temperature of antimatter, which is determined by the antiobserver as being positive, seems negative to us. Negative temperatures make prolonged interactions of matter and antimatter impossible and the paucity of antimatter in our world is thus not surprising. The thermodynamic interactions of matter and antimatter have a profound influence on the perceived direction of time, on dominance of matter over antimatter and on unique properties of the universe at its origin.

References

[1] L. Boltzmann. On certain questions of the theory of gases. *Nature*, 511(1322):413–415, 1895.

[2] L. Boltzmann. *Lecures on gas thoery.* English translation by S.G. Brush. University of California Press, Berkeley and L.A., 1964 (1895,1897).

[3] R. Penrose. *Road to Reality: A Complete Guide to the Laws of the Universe.* A.A. Knopf, N.Y., 2005.

[4] R. P. Feynman. The theory of positrons. *Phys. Rev.*, 76:749–759, Sep 1949.

[5] S. Downes, B. Dutta, and K.Sinha. Attractors, universality and inflation. arXiv: 1203.6892 [hep-th], 2012.

[6] J. Dunning-Davies. Thermodynamics of antimatter vis Santilli's isodualities. *Foundations of Physics Letters*, 12(6):593–599, 1999.

[7] S. W. Hawking. Black holes and thermodynamics. *Phys. Rev. D*, 13(2):191–197, 1976.

[8] H. Price. *Time's Arrow and Archimedes' Point: New Directions for the Physics of Time*. Oxford Univ. Press, Oxford, UK, 1996.

[9] A.Y.Klimenko. Teaching the third law of thermodynamics. *Open Thermodynamic Journal*, 6:1–14, 2012, (arXiv:1208.4189 [physics.class-ph]).

[10] N. F. Ramsey. Thermodynamics and statistical mechanics at negative absolute temperatures. *Phys. Rev.*, 103:20–28, Jul 1956.

[11] L. D. Landau and E. M. Lifshits. *Statistical physics*, volume 5. Pergamon Press, Oxford ; Sydney, N.S.W, 1980.

[12] D. Lynden-Bell. Negative specific heat in astronomy, physics and chemistry. *Physica A: Statistical Mechanics and its Applications*, 263(1):293–304, 1999.

[13] M. Schmidt, R. Kusche, T. Hippler, J. Donges, W. Kronmuller, B. von Issendorff, and H. Haberland. Negative heat capacity for a cluster of 147 sodium atoms. *Physical review letters*, 86(7):1191–1194, 2001.

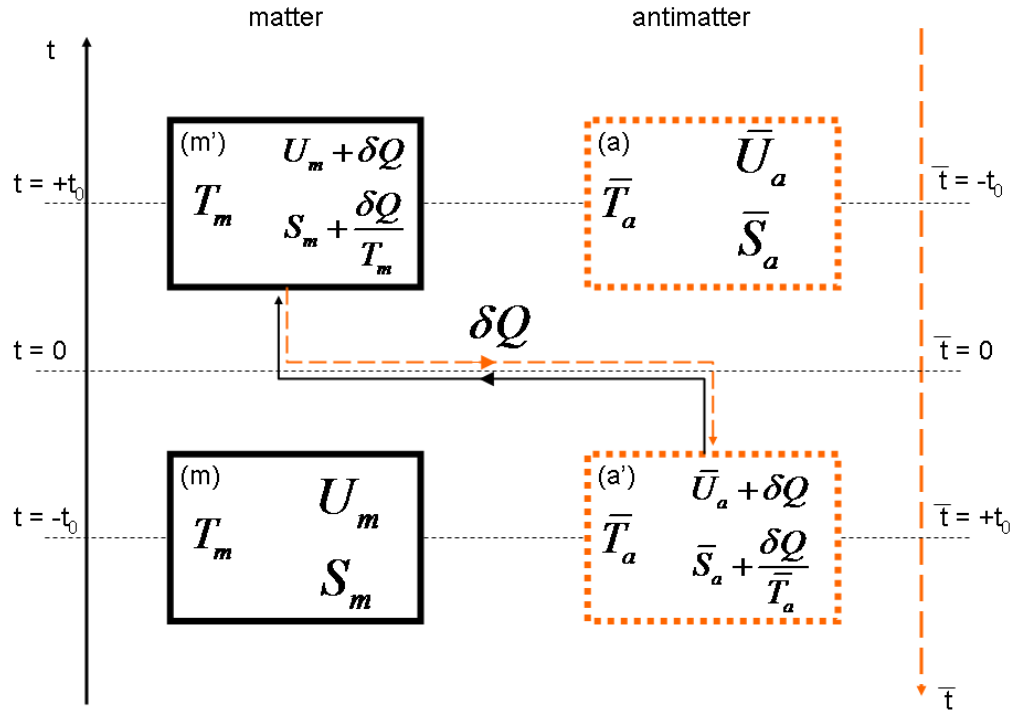[14] A. Y. Klimenko. What is mixing and can it be complex? *Physica Scripta*, T149:to appear, 2012.

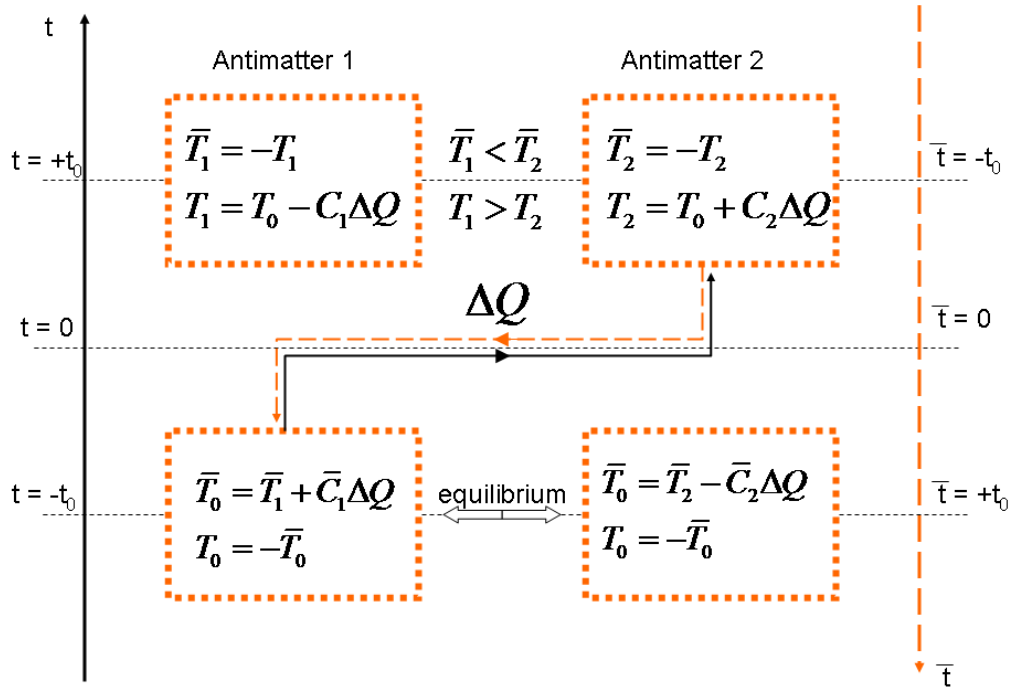Figure 1: Thermodynamic interaction of matter and animatter.

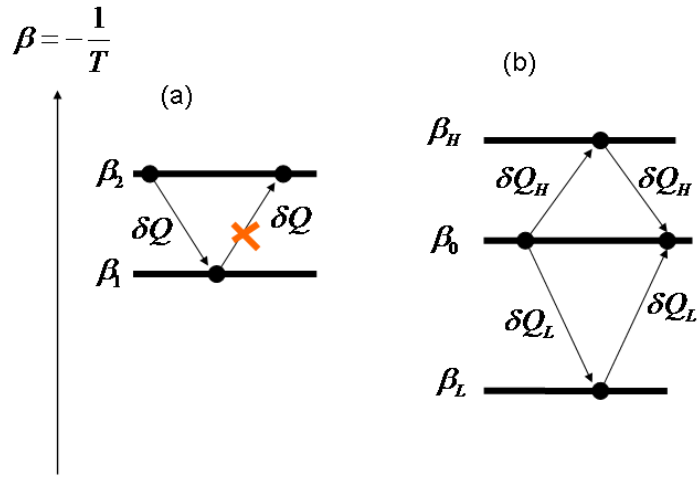Figure 2: Antimatter heat transfer as seen by antiobserver and by us.

Figure 3: Change in qualities of energy a) irreversible and impossible, b) reversible if the overall quality of energy is preserved.
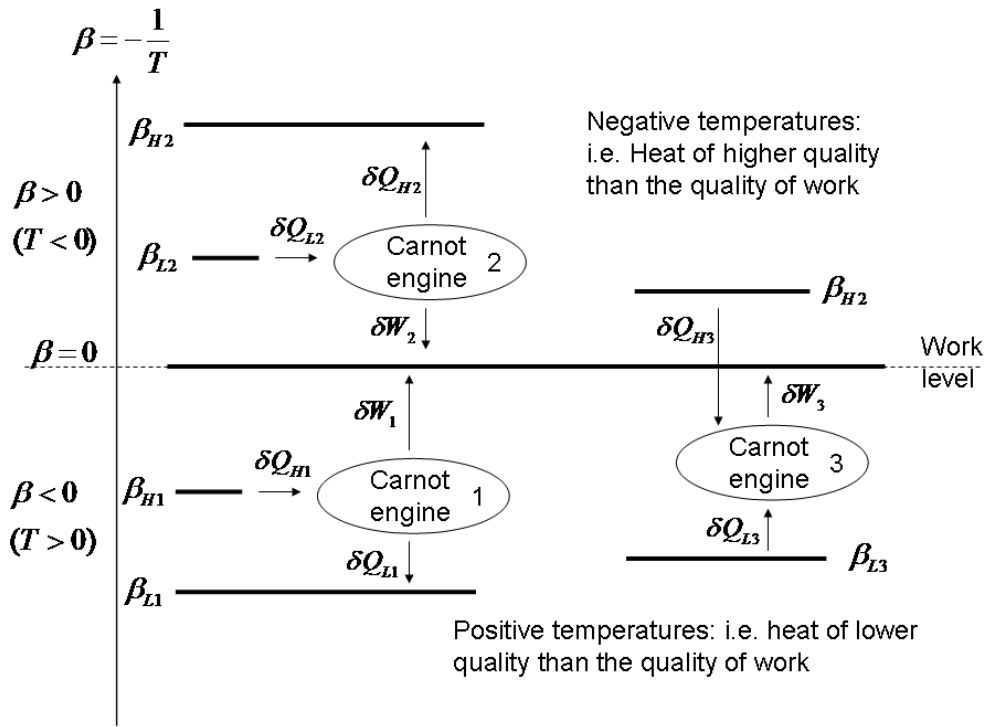
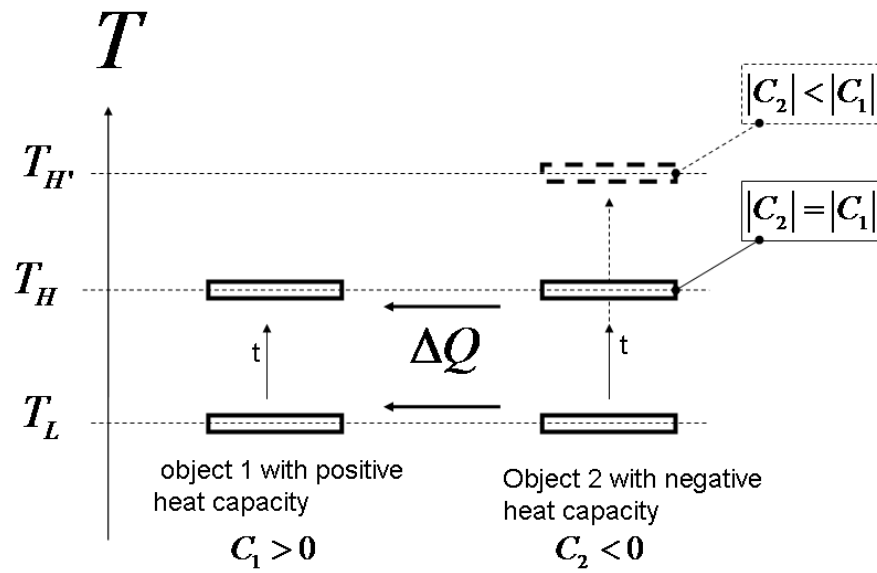Figure 4: Three regimes for ideal reversible (Carnot) engine.

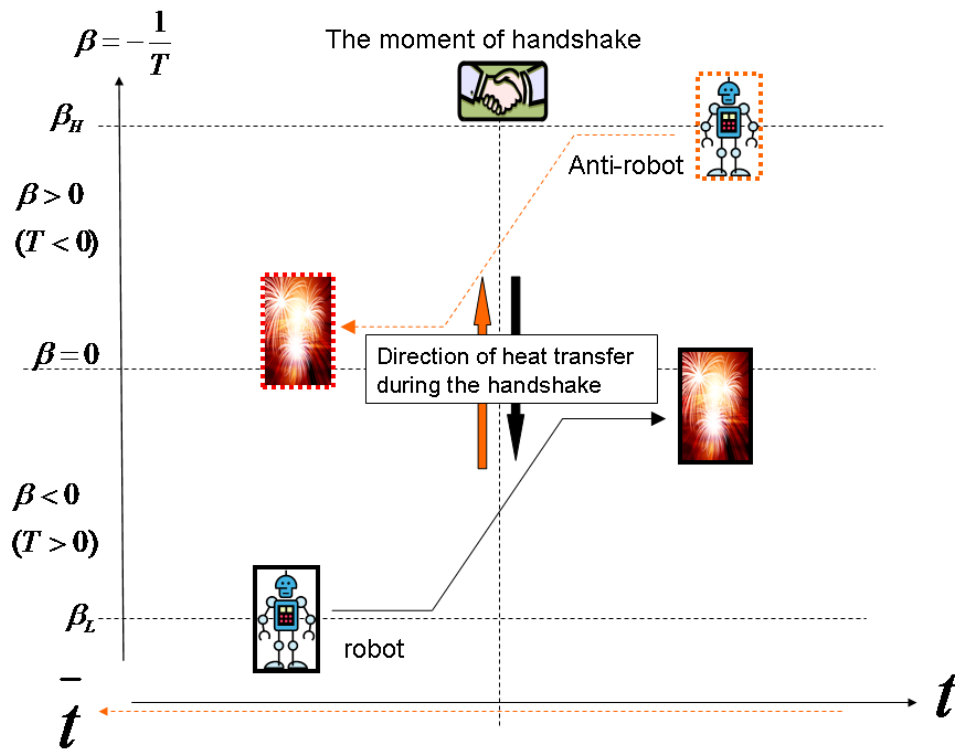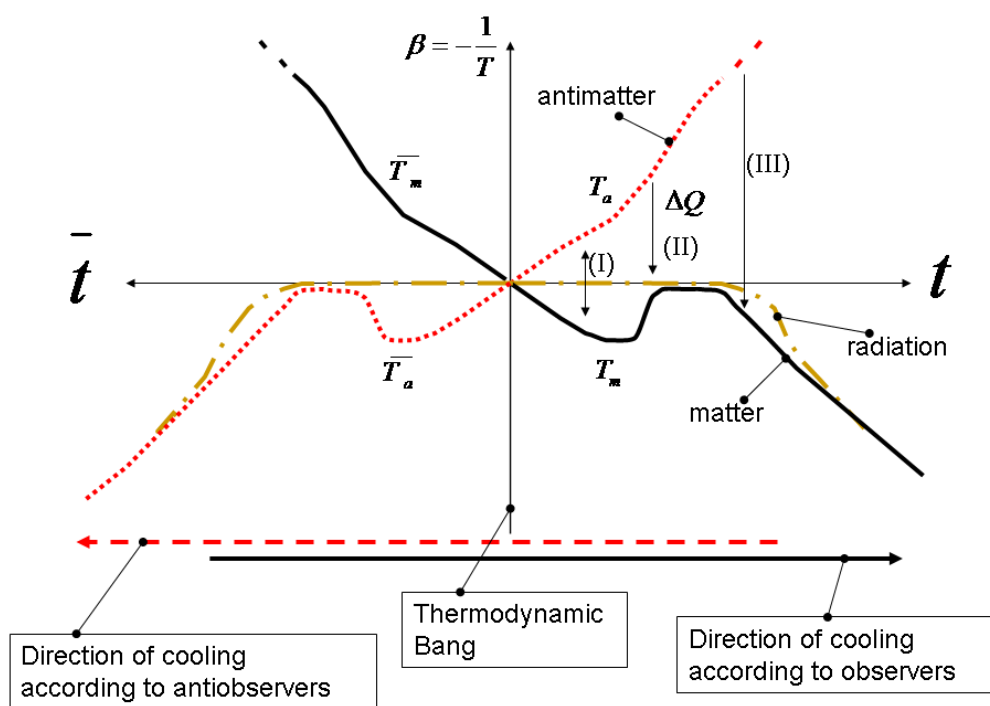Figure 5: Heat transfer when heat capacity might be negative.

Figure 6: Robot and antirobot shake hands.

Figure 7: Thermodynamic Bang